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MODERN COMMERCIAL ARITHMETIC

MODERN

COMMERCIAL ARITHMETIC

BY

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Part I

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PREFACE.

THIS book does not presume to be a treatise on Arithmetic. It contains the Notes of Lectures given by the Author during a series of years in connection with the Training Courses for Teachers of Commercial Arithmetic engaged in the Technical Schools of Yorkshire. It is assumed that students will have had a preparatory course in the fundamental operations of Arithmetic before attempting to work through this volume.

Part I contains all that is required for the Elementary Stage for Commercial Arithmetic. The examples are, in the main, original, though some have been collected from Examination Papers. The method used for decimalisation of money will give results with an error of less than a farthing. It is hoped in the near future to publish Part II. of the Book in which the subjects treated in Part I. will be more fully developed. The Author trusts that the present volume may be found useful by Teachers of Commercial Arithmetic in the numerous Technical Schools of the country and by Teachers of the Commercial Side of the Secondary Schools of the country.

GEO H. DOUGLAS.

THE GRAMMAR SCHOOL.

BRADFORD, 1906

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CHAPTER I

ADDITION AND SUBTRACTION

To add numbers together is to find a single number equivalent to the numbers jointly, and the result is known as the sum, e.g., the sum of 5, 10, and 2 is 17. Quantities can only be added when they are of the same kind; the sum of 5 shillings and 3 shillings is 8 shillings; but we cannot add together 5 shillings and 3 feet. We may add together 5 shillings and 3 pence by first expressing them in the same denomination, thus 60 pence and 3 pence have a sum 63 pence, or 5 shillings and $\frac{1}{4}$ shilling have a sum $5\frac{1}{4}$ shillings. From this rule we get the usual convenient arrangement of numbers or quantities, so that those of the same kind come in the same columns.

EXAMPLES :—

(1) Find the sum of 349, 211, 5014, and 38.

349
211
5014
38
5612

The units have been arranged underneath each other, the tens in the second column, and so on. The mental operation should be as follows: add up the units column thus—8, 12, 13, 22, i.e., 2 tens and 2 units. We put the 2 down and carry the 2 tens along with the other tens thus: 2, 5, 6, 7, 11, i.e., 11 tens or 1 hundred and 1 ten; we put the 1 down in the tens column and proceed similarly with the hundreds column; 1, 3, 6, and so on.

(2) Add together £5 3s. $6\frac{1}{2}$d., £17 12s. 8d., £27 5s. $3\frac{1}{4}$d., and £10 11s. 5d.

£	s.	d.
5	3	$6\frac{1}{2}$
17	12	8
27	5	$3\frac{1}{4}$
10	11	5
£60	12	$10\frac{3}{4}$

Here we have arranged the £'s in one column, the shillings in another, and the pence in a third.

(3) Add together 1·35, ·0056, 1200, 44·75.

$$\begin{array}{r@{}l} 1 & \cdot 35 \\ & \cdot 0056 \\ 1200 & \\ 44 & \cdot 75 \\ \hline 1246 & \cdot 1056 \\ \hline \end{array}$$

(4) Add $2\frac{1}{2}$, $13\frac{3}{4}$, $1\frac{11}{16}$, $22\frac{7}{12}$.

Here we have to change the fractions to equal fractions all of the same name; the smallest denomination to which we can bring halves, fourths, sixteenths, and 12ths is 48ths. We proceed thus:—

$$\begin{aligned} & 2\tfrac{1}{2} + 13\tfrac{3}{4} + 1\tfrac{11}{16} + 22\tfrac{7}{12} \\ &= 38 + \tfrac{24}{48} + \tfrac{36}{48} + \tfrac{33}{48} + \tfrac{28}{48} \\ &= 38 + \tfrac{121}{48} \\ &= 40\tfrac{25}{48} \end{aligned}$$

We have first added the whole numbers together, then changed the fractions to equivalent fractions with 48 for denominator and added the fractions. The sum of the fractions is $\frac{121}{48}$, which is equivalent to 2 units and $\frac{25}{48}$.

In practice the work is often abbreviated thus:—

$$\begin{aligned} & 2\tfrac{1}{2} + 13\tfrac{3}{4} + 1\tfrac{11}{16} + 22\tfrac{7}{12} \\ &= 38\frac{24 + 36 + 33 + 28}{48} \\ &= 40\tfrac{25}{48} \end{aligned}$$

The student should notice the same principle in the various examples, viz., to add together only quantities of the same name. Addition is thus seen to be a simple operation, but it is a very important one in Commercial work. The two essentials are accuracy and speed: the latter can only be acquired by practice. Examples like the following will be found useful.

Add the following numbers both vertically and horizontally :—

561	33	9046	392	
44	9412	5123	123	
357	376	365	456	
1205	28	5	789	
369	472	432	26	
7	5665	1673	525	
804	311	888	674	
3721	2814	706	37	
9420	7777	45	904	
365	4040	5126	12	

The results should be filled into the blank spaces, and the final sum should be the same whether the column of answers or the row of answers be added up.

It is useful in adding to observe quickly the numbers whose sum is 10, e.g., $2+8$, $3+7$, $1+4+5$, $6+4$, &c.

SUBTRACTION.

This is the inverse operation to addition and has a similar principle, i.e., you can only subtract a quantity from another quantity of the same name. To subtract 5 from 12 is to find the number which must be added to 5 to make 12, i.e., 7.

EXAMPLES :—

(1) Subtract 8345 from 9028.

$$\begin{array}{r} 9028 \\ 8345 \\ \hline 683 \\ \hline\hline \end{array}$$

The mental operation is as follows :—5 and 3, 8, put down 3 ; 4 and 8, 12, put down 8 and carry 1 ; 4 and 6, 10, put down 6 and carry 1 ; 9 and 0, 9.

(2) Subtract £23 17s. $2\frac{1}{2}$d. from £39 3s. $6\frac{1}{4}$d.

£	s.	d
39	3	$6\frac{1}{4}$
23	17	$2\frac{1}{2}$
£15	6	$3\frac{3}{4}$

The mental operation is as follows: 2 farthings and 3 farthings, 5 farthings, put down 3 farthings and carry 1d.; 3d. and 3d., 6d., put down 3d.; 17s. and 6s., 23s., put down 6s. and carry £1; 4 and 5, 9, put down 5; 2 and 1, 3, put down 1

(3) Subtract ·075 from 1·2.

$$\begin{array}{r} 1{\cdot}2 \\ {\cdot}075 \\ \hline 1{\cdot}125 \end{array}$$

5 and 5, 10; 8 and 2, 10; 1 and 1, 2; 0 and 1, 1.

(4) $15\frac{2}{3}-3\frac{1}{4}$.

$$15\tfrac{2}{3}-3\tfrac{1}{4}$$
$$=12\tfrac{8-3}{12}$$
$$=12\tfrac{5}{12}$$

(5) Take the sum of 4531, 2918, 337 and 4605 from 62345.

$$\begin{array}{r} 62345 \\ \hline 4531 \\ 2918 \\ 337 \\ 4605 \\ \hline 49954 \end{array}$$

5, 12, 20, 21 *and* 4, 25: put down 4 and carry 2; 2, 5, 6, 9 *and* 5, 14: put down 5 and carry 1; 1, 7, 10, 19, 24 *and* 9, 33: put down 9 and carry 3; 3, 7, 9, 13 *and* 9, 22: put down 9 and carry 2; 2 *and* 4, 6: put down 4. We have thus performed the two operations of adding and subtracting together.

(6) Find change out of £5 after paying away 3s. 3d., 2s. $10\frac{1}{2}$d., 10s. 6d., $7\frac{1}{2}$d. and 13s. 9d.

£5	0	0
0	3	3
0	2	$10\frac{1}{2}$
0	10	6
0	0	$7\frac{1}{2}$
0	13	9
£3	9	0

+ is the sign of the operation of addition.
— is the sign of the operation of subtraction.

EXAMPLES I

1. Add together 34,125, 9,078, 10,433, 57, 819.

2. If 5s. 10d. is spent on gloves, 9s. 6d on collars, and 4s. 8d. on ties, how much is spent altogether ?

3. A country contains 40,502 miles of railway at the beginning of the year, and 49,271 miles at the end How many miles have been added during the year ?

4. The population of Scotland was 1,942,717 in 1891, and 2,173,755 in 1901, what is the increase in the ten years ?

5. In 1893 an ounce of silver was worth 35·625 pence; in 1900 the price had dropped 7·375 pence; what was then the price ?

6. A reservoir contained 23,763,479 gallons of water, and in a dry season when no more water came in 13,682,972 gallons were drawn off. How many gallons were then left ?

7. Four jars contain 7·26 litres, 3·85 litres, 10·7 litres, and 0·90 litres respectively How many litres is this altogether ?

8. A flat is let for £75 a year, and the landlord pays rates and taxes amounting to £26 13s. 9d. What is the net amount he receives ?

9. A cask weighs when empty 4·38 kilogrammes, and when full of oil 50·46 kilogrammes. What does the oil weigh ?

10. From 47,593 take the sum of 6,394, 2,019, 375, 7,426, 10,234, and 4,293.

11. Add the following vertically and horizontally :—

347	756	214	73504	53246	
1423	2334	87	81265	72821	
981	1991	468	4792	90543	
76	645	23	31265	4987	
4502	9	8127	309	759	
813	3306	645	44772	8237	
7614	712	31	56653	93	
365	8104	2244	14151	6298	
99	936	372	76	78458	
1001	816	2815	8034	9367	
4	75	93	7452	48126	
569	2027	3886	38596	5280	
33	415	9425	472	813	
7128	3333	871	6413	4321	
6336	8106	4330	20497	66183	
444	574	2859	81365	4771	
5263	61	9268	426	5042	

12. The two given columns represent the Dr. and Cr. columns of a Journal. Their totals should agree. Test this by adding both up.

Dr.			Cr.		
			30578	5	10
1516	9	4			
25	2	6			
4575	0	0			
6570	8	4			
7690	9	2			
3201	2	6			
4090	10	9			
2000	0	0			
909	3	3			
			2640	3	9
			5940	6	4
			10500	0	0
137	16	2			
			137	16	2
175	6	4			
			21	5	4
			34	3	6
			26	2	2
			32	6	8
			58	8	8
£					

13. The following is a statement showing the value of declared exports. You are to calculate the differences, and insert them in the columns headed increase or decrease, as is done in the first two, and find net increase or decrease and totals.

ARTICLES.	AUG., 1902.			AUG., 1901.			INCREASE.			DECREASE.		
	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
Dress Goods	16146	6	2	18023	11	8				1877	5	6
Linings . .	28256	0	7	17433	15	7	10822	5	0			
Wool ..	34693	6	4	22142	12	2						
Cotton Cloths	17258	18	7	14277	6	2						
Alpaca Hair ..	14049	11	8	3533	19	8						
Machinery . .	7037	17	10	1908	4	7						
Cotton Yarns .	6835	6	4	2262	12	11						
Silk Yarns .	4831	5	10	4473	2	9						
Worsted Coatings .	4676	3	4	5868	4	1						
Silk Goods . .	3714	2	8	8355	12	5						
Iron Wire .	1586	19	0	2365	1	3						
Hides . . .	1010	7	10	582	11	8						
Woollen Cloths	859	8	11	1055	3	8						
Horses	475	0	0	34	10	4						
Chemicals .	138	17	0	468	16	1						
Grease . .	93	17	2	349	18	3						
Paper . ..	453	5	2	110	2	8						

CHAPTER II

MULTIPLICATION

Consider the following example of the ordinary method of multiplying two numbers together.

To multiply 342781 by 235.

WORKING.	EXPLANATION.
342781	
235	
1713905	= 5 times 342781
1028343	= 30 " "
685562	= 200 " "
80553535	= 235 times 342781

The point to be observed is the position of the figures in the three partial products. It is evident that we could have the three partial products in any order, provided that they have their relative positions—for example let us take the 200 times first.

342781	
235	
685562	= 200 times 342781
1028343	= 30 " "
1713905	= 5 " "
80553535	= 235 times 342781

The latter method is preferred by many workers now, and we shall find it useful later in discussing approximations.

Sometimes special relations among the figures of the multiplier will suggest a shortening of the labour.

(1) If the multiplier has 1 as a digit, we may begin with that digit, taking care to give the other partial products their correct relative positions

53426×217

$$\begin{array}{rl}
53426 & \\
106852 & (200 \text{ times}) \\
373982 & (7 \text{ times}) \\
\hline
11593442 & \\
\hline\hline
\end{array}$$

To multiply $3645{\cdot}23$ by $(1{\cdot}02)^2$.

$$\begin{array}{ll}
3645{\cdot}23 & \\
72{\cdot}9046 & (\cdot 02 \text{ times}) \\
\hline
3718{\cdot}1346 & (1{\cdot}02 \text{ times}) \\
74{\cdot}362692 & (\cdot 02 \text{ of } 1{\cdot}02 \text{ times}) \\
\hline
3792{\cdot}497292 & \\
\hline\hline
\end{array}$$

(2) 632041×192328.

We notice that 32 is 4 times 8 and 192 is 6 times 32: we may perform the multiplication with three partial products.

$$\begin{array}{rlr}
632041 & & \\
192328 & & \\
\hline
5056328 & = & 8 \text{ times} \\
20225312 & = 40 \times 8 \text{ times} = & 320 \;\;,, \\
121351872 & = 600 \times 320 \;\;,, \;\; = & 192000 \;\;,, \\
\hline
121559181448 & & 192328 \\
\hline\hline
\end{array}$$

Again:

$$\begin{array}{rlr}
392056 & & \\
14735 & & \\
\hline
2744392 & \phantom{20 \times{}}700 \text{ times} = & 700 \text{ times} \\
5488784 & 20 \times 700 \;\;,, \;\; = & 14000 \;\;,, \\
13721960 & 5 \times 7 \;\;\;\;\;,, \;\; = & 35 \;\;,, \\
\hline
5776945160 & & 14735 \;\;,, \\
\hline\hline
\end{array}$$

In the latter, for example, we multiplied by 7 first; we next took twice that 7 line and placed it one place to the left; finally we took 5 times the 7 line, placing the figures two places to the right of the 7 line.

(3) As $2 \times 5 = 10$, $4 \times 25 = 100$, $8 \times 125 = 1000$, we can multiply by 5 by adding a cipher and dividing by 2; multiply by 25 by adding two ciphers and dividing by 4; multiply by 125 by adding 3 ciphers and dividing by 8.

$$\begin{aligned} 136 \times 125 &= 136000 \div 8 = 17000 \\ 2013 \times 125 &= 2013000 \div 8 = 251625 \\ 375 \times 25 &= 37500 \div 4 = 9375 \end{aligned}$$

In practice it is done mentally.

$$\begin{aligned} 243 \times 125 &= 30375 \\ 6{\cdot}13 \times 125 &= 766{\cdot}25 \\ 36{\cdot}12 \times 25 &= 903 \end{aligned}$$

In the last example we divide by 4 moving the figures two places to the left: thus $36 \div 4 = 9$ and the 100 makes this 900.

(4) If the multiplier consists entirely of nines the following method can be used.

$$\begin{array}{rr} 81475 \times 999 \text{ Write down} & 81475000 \\ \text{and subtract} & 81475 \\ \hline \text{Result} = & \underline{81393525} \end{array}$$

This is derived from the fact that $999 = 1000 - 1$. A similar method may be used when the multiplier is near a power of ten.

25637×9996:

$$\begin{array}{rrr} 256370000 = & 10000 & \text{times } 25637 \\ 102548 = & 4 & \text{times } 25637 \\ \hline \underline{\underline{256267452}} = & \underline{\underline{9996}} & \underline{\underline{\text{times } 25637}} \end{array}$$

Such products can be obtained mentally.

(5) Multiplication by 11.

$$62957 \times 11 = 692527$$

Mental process, put down 7.

7 and 5 = 12,	put down	2,	carry 1.
1 + 5 + 9 = 15,	,,	5,	,, 1.
1 + 9 + 2 = 12,	,,	2,	,, 1.
1 + 2 + 6 = 9,	,,	9.	
6 = 6,	,,	6.	

The reason will be seen if the following be examined.

$$\begin{array}{r} 62957 \\ 11 \\ \hline 62957 \\ 62957 \\ \hline 692527 \end{array}$$

The object of practising these short methods is to beget in the student the alertness to notice and take advantage of anything special in the figures.

Example of multiplication in decimals.

$$37 \cdot 654 \times 20 \cdot 143.$$

$$\begin{array}{r} 37 \cdot 654 \\ 20 \cdot 143 \\ \hline 112962 \\ 150616 \\ 37654 \\ 75308 \\ \hline 758 \cdot 464522 \end{array}$$

This is the usual method; the numbers are multiplied without reference to the decimal point; the number of figures after the point in both multiplier and multiplicand give the number of decimal figures in the product—$3+3$ or 6 in the present example. The student should notice the partial product by 2 in the above example; we have moved two places to the right from the previous partial product.

Before showing another method there are two preliminary points to be considered.

(1) Remember the effect of multiplication or division by a power of ten—each figure is simply moved up in the former and moved down in the latter as many places as there are ciphers in the operator; e.g.,

$$364 \times 100 = 36400$$
$$43 \cdot 475 \times 100 = 4347 \cdot 5$$
$$\cdot 00576 \times 1000 = 5 \cdot 76$$
$$12 \div 10 = 1 \cdot 2$$
$$\cdot 3456 \div 100 = \cdot 003456$$
$$9376 \cdot 8125 \div 10000 = \cdot 93768125$$

The result is the same if we consider the point moved in the reverse direction.

(2) If we multiply one factor and divide the other factor by the same number the product will be the same; e.g.,

$$72 \times 12 = 864$$
$$216 \times 4 = 864$$
(We have multiplied 72 by 3 and divided 12 by 3.)
$$18 \times 48 = 864 \ (18 = 72 \div 4, \ 48 = 12 \times 4).$$

Similarly $36{\cdot}478 \times {\cdot}356$ will be the same as $3647{\cdot}8 \times {\cdot}00356$, as we have taken 100 times 36·478 and 100th part of ·356. It will also be the same if we take $3{\cdot}6478 \times 3{\cdot}56$.

Thus the other method I wish to show is begun thus: make the *multiplier* have one unit figure by moving the point; in the multiplicand move the point as many places the reverse way.

Example: $37{\cdot}654 \times 20{\cdot}143$.

```
376·54
  2·0143
----------
753·08
  3·7654
  1·50616
   ·112962
----------
758·464522
```

Compare this with the previous working. The result is the same. In this example we have begun the multiplication with the 2 or the unit figure. We can easily tell at once where our decimal point is to be as the product of $2 \times {\cdot}04$ must be ·08. This is a great advantage, as we shall see in approximate multiplication. This method should be practised.

Another example: $3048{\cdot}725 \times {\cdot}04236$.

```
 30·48725
  4·236      One unit figure
------------
121·94900
  6·097450
   ·9146175
   ·18292350
------------
129·14399100
```

The original factors had 3 + 5 decimal figures. The product has 8.

To multiply a sum of money by a whole number, e.g., £27 15s. 8½d. × 173.

The following is a compact way of showing the working.

£27 15 8½	
173	
£4806 17 6½	

WORKING.

2 | 173 halfpence.

86d. + ½d.
1384 = 8 × 173d.

12 | 1470

122s. + 6d
865 = 5 × 173 sh.
173 = 10 × 173sh.

20 | 2717

135£ + 17sh.
1211 = 7 × £173
346 = 20 × £173

£4806

In practice all that need be shown is :—

£35 12 7¼
23

£819 9 10¾

4 | 23

5—3
161

12 | 166

13—10
46
23

20 | 289

14—9
115
69

819

If the multiplier is small the work may be done mentally.

£13 17s. 6¼d × 7 = £97 2s. 7¾d.

Sometimes the multiplier may be split up into factors.

£17	3	2½	× 35 (factors 7 & 5).
		7	
120	2	5½	
		5	
£600	12	3½	

EXAMPLES II

1. Find the following products, beginning with the left-hand figures of the multiplier —371156 × 205003, 4289 × 371, 14562 × 79, 3769 × 3769, 2046812 × 3507, 1475 × 203, 61275 × 417.

2. Find, with as little working as possible, 84375 × 99, 37621 × 998, 12345 × 798, 505412 × 9993, 39256 × 11, 6523 × 11, 34756 × 25, 803·4 × 25, 2·475 × 25, 81234 × 125, 675 × 1·25, 3·475 × 125.

3. Write down the following products :—3·4765 × 1000, ·0459 × 100, 1·0101 × 10, 2·6783 × ·1, 89·341 × ·01, 23 × ·0001, ·96845 × 10000, 2·375 × 11, 1·475 × 1·1, 675 × 125, $(125)^2$, 625 × ·25, 3·333 × 1·25.

4. Multiply £3 17s. 6d. by 3, £215 2s. 7½d. by 8, £111 11s. 1d. by 67, £201 10s. 6d. by 33, £9 19s. 2¾d by 107, £614 13s. 7d. by 41.

5. What is the price of 30 copies of a book at 2s. 8d. each ?

6. How far does a train go in 8 hours at 43 miles an hour ?

7. The average weight of 9 persons is 1 cwt. 34 lbs., what is the weight of the nine together ?

8. If I employ a man at a weekly wage of £1 0s. 6d., how much do I pay him in a year (52 weeks) ?

9. Find the total weight of 15 trucks with their contents, each truck weighing 2 tons 3 cwt., and containing 6 tons 2 qrs.

10. Find the wages for a week's work of 44 hours at 10d. an hour.

11. A contracts to supply B daily (Sundays excepted) with goods worth 13s. 6d., and B similarly supplies A with goods worth 11s. 3d. If the contract extends from January 1st, 1904, to 31st March, 1904, how much did B owe A on 31st March ?

12. If each book of a series weighs 0·32 kilogram, what will 40 of them weigh ?

13. Find the following products, in each case making the multiplier into a number with one unit figure :—$37{\cdot}025 \times {\cdot}0456$, $104{\cdot}378 \times 3{\cdot}275$, ${\cdot}84367 \times 42{\cdot}321$, $8{\cdot}8723 \times {\cdot}00037$, ${\cdot}5635 \times {\cdot}322$, $64512 \times 1{\cdot}035$, $32{\cdot}456 \times 20{\cdot}3004$.

14. The following lots of land were sold at the prices given. Find the total amount realised 17,822 sq. yds at 4s. 2d., 14,531 sq. yds. at 3s. 6d., 10,531 sq. yds at 1s 1d., 5,491 sq. yds. at 1s. 7d., 26,079 sq. yds. at 11d.

15. Multiply £234 17s 5d. by 3012.

CHAPTER III

DIVISION

The process of division, i.e., of finding how often one number is contained in another larger number, is a familiar one to most workers in arithmetic. Below are two worked examples. When the divisor is small, short division should be used, and the student should practise by divisors up to 20.

Divide 363427 by 7.

$$
\begin{array}{l}
7\overline{)363427} \\
\quad 51918 \text{ and remainder } 1. \\
\text{or, } 51918\tfrac{1}{7}
\end{array}
$$

Divide 17116122 by 347.

$$
\begin{array}{r}
347)17116122(49326 \\
\underline{1388} \\
3236 \\
\underline{3123} \\
1131 \\
\underline{1041} \\
902 \\
\underline{694} \\
2082 \\
\underline{\underline{2082}}
\end{array}
$$

If the divisor is a number such as 20, 300 or 7000, the following method should be used :—

$$
\begin{array}{l}
20\overline{)43124} \\
\quad 2156 \text{ and } 4 \text{ over}
\end{array}
$$

Use 2 as mental divisor, putting the result one figure (because of the ten) further to the right.

$$7000\overline{)83124}$$
$$11 \text{ and } 6124 \text{ over}$$

The figures of the quotient are placed three places to the right as above: 7 goes into 8, once; move the 1 to under the 2.

A decimal can be divided by a whole number very conveniently thus when short division is possible.

$$5\overline{)23 \cdot 125} \qquad 600\overline{)41 \cdot 232}$$
$$4 \cdot 625 \qquad 0 \cdot 06872$$

Difficulty is often found in placing the decimal point in the quotient when the divisor is a decimal. Various methods are used to reduce the difficulty. One very common method is shown below.

Divide ·38691 by ·027.

Multiply both by 1000, producing the question:—Divide 386·91 by 27. The 1000 was chosen because it converts the divisor, the operator, into a whole number.

```
     14·33
27)386·91(
   27
   ---
   116
   108
   ---
     89
     81
     --
      81
      81
```

The advantage of having a whole number for divisor is that the figures of the quotient will be of the same denomination as the figures of the corresponding part of the dividend.

We first divide 380 by 27, giving an answer 10, so that as the 8 is two places before the point, so also the 1 of the quotient is two places before the point. A convenient arrangement of the quotient is to put the figures of the quotient above the corresponding figures of the dividend, as in above example.

Example :—Divide ·1387 by 7·3.

```
       ·019
  73)1·387(
      73
      ---
       657
       657
       ===
```

The student might also consider the following method :—
Divide ·38691 by ·027.

Make the divisor a number of one unit figure.

```
        14 33
  2·7)38·691(
      27
      ---
      116
      108
      ---
        89
        81
       ---
        81
        81
       ===
```

To fix the point in the answer we divide roughly by the unit figure only, in above case by the 2. The 2 divides the 3 and thus we see that the first figure of quotient is two places before the point. We then proceed as in simple division without considering the point. We shall find this method useful in contracted division of decimals.

The figuring of long division may be shortened by using the method known as Italian Division.

```
347)17116122(49326
      3236
       1131
         902
         2082
            0
         ====
```

The multiplication and subtraction are worked simultaneously thus : 4 × 7, 28 and 3, 31. Put down 3 and carry 3, 4 × 4, 16 and 3, 19 and 2, 21 ; put down 2 and carry 2, 4 × 3, 12 and 2, 14 and 3, 17. Put down 3. Take down next figure

of the dividend, 6. 9×7, 63 and 3, 66 ; put down 3 and carry 6 ; 9×4, 36 and 6, 42 and 1, 43 ; put down 1 and carry 4 ; 9×3, 27 and 4, 31 and 1, 32 ; put down 1. Proceed similarly with the other figures of the quotient. The student should work through the example for himself. This method will be used in all future examples.

Divide 31·482 by 79·5.

$$\begin{array}{r l}
 & \cdot 396 \\
7{\cdot}95 \big) & 31482 \big(\text{Quotient } \cdot 396 \\
 & 7632 \\
 & \;\;4770 \\
 & \;\;\;\;000
\end{array}$$

Similar methods can be used in Compound Division.

$$\begin{array}{r r r r l r r r}
 & \pounds & \text{s} & \text{d.} & & \pounds & \text{s.} & \text{d.} \\
207\big) & 3002 & 11 & 6\frac{3}{4} & \big(& 14 & 10 & 1\frac{1}{4} \\
 & 207 & & & & & & \\
\hline
 & 932 & & & & & & \\
 & 828 & & & & & & \\
\hline
 & 104 & & & & & & \\
 & 20 & & & & & & \\
\hline
 & 2091 & & & & & & \\
 & 2070 & & & & & & \\
\hline
 & 21 & & & & & & \\
 & 12 & & & & & & \\
\hline
 & 258 & & & & & & \\
 & 207 & & & & & & \\
\hline
 & 51 & & & & & & \\
 & 4 & & & & & & \\
\hline
 & 207 & & & & & & \\
 & 207 & & & & & &
\end{array}$$

or,

$$\begin{array}{r r r r l r r r}
 & \pounds & \text{s.} & \text{d.} & & \pounds & \text{s.} & \text{d.} \\
207\big) & 3002 & 11 & 6\frac{3}{4} & \big(& 14 & 10 & 1\frac{1}{4}. \\
 & 932 & & & & & & \\
 & 104 & & & & & & \\
\hline
 & 2091 & & & & & & \\
 & 21 & & & & & & \\
\hline
 & 258 & & & & & & \\
 & 51 & & & & & & \\
\hline
 & 207 & & & & & & \\
 & 0 & & & & & &
\end{array}$$

In the second form the figuring is cut short. The last remainder after division of the £'s is 104 ; this is multiplied by 20 to convert them to shillings and the 11s. are added, giving 2091 as the number of shillings to be divided. Similarly the 21 remaining shillings are multiplied by 12 and the 6d. added, giving 258d.

Questions like the following can be worked by division :—How often is 17s. 6d. contained in £14 ? We reduce each to the same name, e.g., to pence.

```
                  £14
                    20
    s.  d.        ---
    17  6         280
        12         12
       210)3360(16
            1260
               0
```

The answer is 16 times.

EXAMPLES III

1. Calculate the following by short division :—343714 ÷ 7, 8412368 ÷ 8, 33·124 ÷ 4, 56745 ÷ 20, 30·125 ÷ 30, 9·37 ÷ 300, 121278 ÷ 17, 66499 ÷ 13, £234 3s. ÷ 6, £87 15s. 5d. ÷ 11, £77 6s. 3d. ÷ 20, £37 9s. 5¼d. ÷ 7.

2. Perform the following divisions :—964374 ÷ 165, 256834 ÷ 219, 8788725 ÷ 201, 1·8941 ÷ 1·3, 71·595 ÷ ·037, ·00700065 ÷ ·0705, 37 82 ÷ 40,000 £373 19s. 7¾d. ÷ 34, £3 7s. 8d. ÷ 112, £14,775 6s. 3d. ÷ 75.

3 A dozen pairs of boots cost £4 15s. 9d.; find the cost of a single pair to the nearest penny.

4. If a cwt. of goods cost £5 16s. 8d , find the price of 1 lb.

5. How many articles, each costing 4s. 7½d., can be purchased with £71 2s. 6d., and how much money will be left over ?

6. A pile of 650 cannon balls weighs 7 tons 16 cwt. 2 qrs. 22 lbs. Find the weight of each ball.

7. How many books at 1s. 1½d. each could be bought for £8 2s.?

8. The weight of a quantity of goods is 6 cwt. 2 qr. 27 lbs. 13 oz. How many parcels, each weighing 5 lbs 7 oz., can be made from them ?

CHAPTER IV

APPROXIMATIONS

It often happens that results of calculations are not required beyond a certain number of figures; and very often the results are not correct to many figures. It may generally be said that few measurements are correct to more than four figures. If we measure a distance and say it is 5·34785 inches, we cannot be sure of the figures beyond 5·34, i.e., the length is between 5·34 and 5·35 inches. If we say 5·34 inches the error is ·00785 of an inch; if we say 5·35 the error is ·00215. Thus in making allowances for figures left out we follow the practice of adding one to the last figure retained if the next figure is 5 or over 5.

Examples: 23·04624 is written 23·046 correct to three decimal places, and 23·05 correct to two decimal places.

Similarly if we wish to multiply 14·2386 by 2 correct to three decimals we get the result 28·477. We allow 1 for the product of 6×2 and say twice 8, 16, add 1, 17. But if we wish the product to two decimal places we get 28·48, allowing 2 from the product 16, i.e., of 8×2.

Decimalization of Money.

For commercial work it is generally sufficient to get the result correct to three decimal places of £'s. This, as we shall see, gives an error less than ¼d.

Method for three decimal places.

Find how many farthings there are in the pence and farthings; put the unit figure in the third decimal place; multiply the number of shillings by 5, add the tens carried from the farthings and put unit figure in second decimal place, tens figure in first decimal place. The number of '£'s is of course the whole number in the result.

Examples: £17 12s. 3d. = £17·612.

3d. = 12f., put down 2 and carry 1.
12 × 5 = 60, add the 1 carried, i.e., 61.
Put 1 in second decimal place, 6 in the first, and prefix the £17. The process can readily be done mentally.

Again: £3 13s. $2\frac{3}{4}$d. = £3·661.

$2\frac{3}{4}$d. = 11f., put down 1 and carry 1
13 × 5 = 65, add the 1, 66.

£456 8s. $4\frac{1}{2}$d. = £456·418.
£24 1s. $1\frac{1}{2}$d. = £24·056.

Here as 1 × 5 gives no tens figure we fill up the vacant first decimal place with 0.

Reasons for the method.

(*a*) $\frac{1}{4}$d. = £$\frac{1}{960}$ = £$\frac{1}{1000}$ nearly = £·001.

Hence we deduce the method of writing the farthings in third decimal place.

(*b*) 1s. = £$\frac{1}{20}$ = £$\frac{5}{100}$ = £·05.

Hence we deduce the method of multiplying the shillings by 5 and putting result with *two* decimal places.

Consideration of the error involved.

The error due to each farthing in pence and farthings.

= £$(\frac{1}{960} - \frac{1}{1000})$ or £$\frac{100-96}{96000}$ or £$\frac{1}{96000}$ or £$\frac{1}{24000}$ too little.

Example: £17 12s. 3d. = £17·612.

The error due to 3d. or 12 farthings = £$\frac{12}{24000}$ or £$\frac{1}{2000}$ or one two-thousandth part of a £. This is not as much as the lowest current coin.

Corollary.

The error due to 6d. is £$\frac{24}{24000}$ or £·001 too little, i.e., $\frac{1}{4}$d. too little.

So we can get the exact decimal when the pence is 6d., by reckoning an extra farthing.

Examples: £77 3s. 6d. = £77·175.

6d. = 24f., say 25f.
£3 12s. 6d. = £3·625.

What shall we do in the case of pence over 6d.? Evidently we should reckon an extra farthing. So that in every case the error will be less than what is due to 6d., i.e., less than $\frac{1}{4}$d.

Examples: £205 17s. 10$\frac{1}{2}$d. = £205·893.

10$\frac{1}{2}$d. = 42f., add 1f. for the 6d. in 10$\frac{1}{2}$d.
£6 6s. 9d. = £6·337.

How to get additional figures when required.

The error due to $\frac{1}{4}$d. is £$\frac{1}{24000}$.

Therefore the error due to 1d. is £$\frac{1}{6000}$ or £·000$\frac{1}{6}$.

We divide the pence and farthings expressed as decimals of 1d. by 6000: or shortly divide by 6, the first figure of quotient (always 0) will be in third decimal place, next figure the fourth decimal place, and so on.

Examples: £17 12s. 3d. = £17·6125.

We first get the result to three decimals, then dividing 3 by 6000, i.e., ·003 by 6, we get ·0005. In practice we say 3 divided by 6 is 0, 30 by 6 is 5, which gives us the fourth figure, and there are no more.

Again: £3 13s. 2$\frac{3}{4}$d. = £3·66145833 · · · · where 3 repeats. In practice we say 2 75 by 6 gives 45833 · · where the 4 is in the fourth decimal place.

Again: £456 8s. 4$\frac{1}{2}$d. = £456·41875.
£24 1s. 1$\frac{1}{2}$d. = £24·05625.
£205 17s. 10$\frac{1}{2}$d. = £205·89375.
£6 6s. 9d. = £6·3375.

In the last two cases, as we corrected for 6d. in getting three places, we divide 4·5 and 3 respectively by 6 for the fourth figure.

Again : £2 15s. $0\frac{1}{2}$d. = £2·7520833 · · · · · · ·

Here we have to divide 0 5 by 6 for the fourth figure, which is evidently 0.

To change a result, correct to 3 decimal places into £ s. d., we reverse the process.

Divide the first two decimal places by 5 ; the result gives shillings ; what is over is reckoned as tens of farthings, so taking the third decimal figure as units of farthings, we get the number of farthings which can easily be turned into pence and farthings.

Examples : £1·172 (correct to 3 decimal places) = £1 3s $5\frac{1}{2}$d. to nearest farthing.

$\frac{17}{5} = 3$ and 2 over. Reckon 3 as shillings the 2 as 20 farthings · thus we have 22 farthings or $5\frac{1}{2}$d.

£3·625 = £3 12s. 6d.

$\frac{62}{5} = 12$ and 2 over. Reckon 12 as shillings, the 2 as 20 farthings. Thus we have 25 farthings or $6\frac{1}{4}$d., but as we added an additional farthing in case of 6d., we now subtract a $\frac{1}{4}$d. giving above result.

£12·408 = £12 8s. 2d.
£27·826875 = £27·827 (to 3 places) = £27 16s. $6\frac{1}{2}$d.
£79·72634 = £79·726 = £79 14s. $6\frac{1}{4}$d.

Examples of application.

Multiply £607 13s. 9d. by $1\frac{5}{8}$ to nearest farthing.

£	
607·6875	1 times
303·8438	$\frac{4}{8}$ or $\frac{1}{2}$ times.
75·9609	$\frac{1}{8}$ (or $\frac{1}{4}$ of $\frac{4}{8}$) times.
987·4922	
= £987 9s. $10\frac{1}{4}$d.	

Divide £77 14s. 2½d. by 64, to nearest farthing.

```
          £
       8 |77·71041
  64     |--------
       8 | 9·7138
         |--------
         | 1·2142 = £1 4s. 3½d.
```

Compare with :

```
            £    s    d
        8 | 77   14   2½
  64      |-------------
        8 |  9   14   3 5/16
          |-------------
             1    4   3 53/128
```

Approximate addition or subtraction.

Write each number correct to the decimal place next after the place required.

Example: Add together 37·0423, 5·0375, 0·001923875, 0·000796, 1·010182 to two decimal places.

Approximately.	*In full.*
37·042	37·0423
5·038	5·0375
0·002	·001923875
0·001	·000796
1·010	1·010182
43·09	43·092701875

Subtract 1·258175 from 24·638975 to two decimals.

```
 24·639
  1·258
 ------
 23·38
```

Approximate Multiplication.

Refer back to Chapter II. on method of multiplication by left hand digit first and how to make operator have one unit figure.

Example: Multiply 47·318275 by ·08324.

We shall first multiply in full. We move the point in multiplier to make it 8·324. We therefore have to move the point in multiplicand two places in reverse direction, i.e., to the left in this case.

```
0·47318275
8·324
-------------
3·78546200
 ·141954825
    94636550
    18927310
-------------
3·93877121100
=============
```

This correct to two places is 3·94 Let us consider how we might have reduced the figuring.

```
0·473|1827
8·324|
-----|
3 785|
  ·142|
     9|
     2|
-----
3·94
====
```

We wish to work to two places; we really work to one place more. Thus having decided to work to three places, we begin multiplying the unit figure 8 into 3, the figure in the third decimal place of the multiplicand. $3 \times 8 = 24$. But we allow for the other figures by saying 8 times 1, 8, carry 1 as it is over 5 and in this way we get 5 in the third place of the first partial product. Then as the next figure 3 in multiplier has one decimal place we begin the multiplication with the figure 7, the second decimal figure in the multiplicand, carrying 1 from 3 times 3, thus we say 3 times 7, 21, and 1 to carry, 22. Thus we have the second partial product. Again we begin the multiplication by 2 with the figure 4 in the multiplicand carrying 1 from 7×2, 14. The multiplication by 4 begins at the unit figure of multiplicand which is 0, but we have 2

to carry from 4×4, 16. When we add the figures in the third decimal place we get 19, so we carry 2 to the next line, as it is over 15.

It is interesting to compare the two workings. In the approximate working it will be seen that each partial product is correct up to three places.

Example : $89{\cdot}1876 \times 64{\cdot}03126$ to nearest integer.

```
891·8|76
  6·4|03126
------|------
5351·2|
 356·7|
   2·7|
     1|
------|
5711· |
```

The first partial product, that by 6, begins at 8, the first decimal place, one place beyond integers, and we carry 4 from 7×6, 42. The product by 4 begins at 1 and 3 is carried from 8×4, 32. The product by 0 begins at 9; but we pass that and begin the product by 3 at the 8, carrying 3 from 9×3, 27. The product by 1 begins at 0 in front of 8, with 1 to carry from 8×1, 8. All the products after this result in 0 so far as the places we are considering, so we stop and add. The figures in the first decimal place add up to 17, for which we carry 2.

A mechanical assistance may be obtained if we place the unit figure of multiplier under the figure in multiplicand where multiplication begins. Each successive figure to the right in multiplier will then begin multiplying the corresponding figure to the left in multiplicand.

Example : $0{\cdot}00719 \times {\cdot}086345$ to 5 places.

```
·000071|9
      8|·6345
-------|------
·000575|
     43|
      2|
-------|
·00062
```

Approximate Division.

Divide 532·718964 by 8·13256 to 3 places.

```
8·13256)532·71896|64(65·504
        487 9536 |
        -------- |
         44 7653 |6
         40 6628 |0
         ------- |
          4 1025 |64
          4 0662 |80
          ------ |
             362 |8400
             325 |3024
             --- |
              37 |53760
```

The above is the ordinary method. We see that the next figure is 4 and so the quotient is 65·504. It will be seen that the figures to the right of the vertical line are not necessary, so that we might have worked as follows:

```
8·13256)532·718964(65·504(4)
        487 9536
        --------
         44 7653
         40 6628
         -------
          4 1025
          4 0663
          ------
             362
             325
             ---
              37
```

Compare each result with the preceding working. Dividing roughly by 8 we see the first figure in the answer is to be 60, that is 6 in the tens place. Thus we know that we shall have two figures before the point, and we wish three decimal figures, i.e., five figures in all. It is enough to keep one more figure in the divisor, i.e., 6 in this example. We then begin with 6 in quotient, giving a remainder, 447653. Instead of taking down another figure, we knock one figure off the divisor, giving us as divisor 81325, five figures, or one more than the number still required in quotient. The next figure in quotient is 5.

We multiply 81325 by 5, carrying 3 from 6 × 5, 30, giving us 406628 with remainder 41025. We knock another figure off the divisor and now use 3132. Multiplying 8132 by 5, carrying 3 from 5 × 5, 25, we have 40663, giving remainder 362. We knock another figure off divisor giving 813 as divisor, and as this goes 0 times in 362 we put 0 in quotient and knock another figure off the divisor, giving us as divisor 81 which goes 4 times. Multiplying 81 by 4 and carrying 1 from 3 × 4, 12, we have 325, leaving a remainder 37 which on mental division by remaining figure of divisor 8 gives 4 as 4th decimal figure; from which we have quotient 65·504.

The work may be still further reduced if the student use complementary subtraction with the multiplication (see Chapter III.).

$$
\begin{array}{r l}
8{\cdot}1\cancel{3}\cancel{2}\cancel{5}\cancel{6}\,\big)\,532\ 718964\big(& 65\ 504 \\
44\ 7653 & \\
4\ 1023 & \\
362 & \\
37 & \\
\hline
\end{array}
$$

Divide 45·3625 by ·07958 correct to 1 decimal place. Make the divisor 7·958 and therefore the dividend 4536·25. When we divide roughly by 7 the first figure of the answer is seen to be three places before the point. We, therefore, need four figures in quotient and hence five in divisor. As we have only four figures in divisor there is no contraction to begin with.

$$
\begin{array}{r l}
7{\cdot}9\cancel{5}\cancel{8}\,\big)\,4536\ 25\big(& 570{\cdot}0(\cancel{2}) \\
557\ 25 & \\
19 & \\
\hline
\end{array}
$$

The contraction begins when the number of figures in the divisor is one more than the number of figures still to be obtained in the quotient.

Divide 5673900 by 447237 to 2 decimal places.

$$
\begin{array}{r l}
4{\cdot}\cancel{4}\cancel{7}\cancel{2}\cancel{3}\cancel{7}\,\big)\,56{\cdot}739\big(& 12{\cdot}68(\cancel{6}) \\
12\ 015 & = 12{\cdot}69 \\
3\ 070 & \\
387 & \\
29 & \\
\hline
\end{array}
$$

Dividing by 4 gives first figure in quotient two places before the point, showing that there will be four figures in quotient; we keep five in divisor by knocking off the 7. At end $29 \div 4 \cdot 4$ gives 6, hence we write answer as 12·69.

If we have to divide money by a large divisor the approximations will give an answer correct to farthings.

Example. The rateable value of a district being £537841 10s., what rate in the £ must be levied (to nearest farthing) to produce at least £25060?

This resolves into dividing £25060 by 537841·5.

$$
\begin{array}{rll}
 & \pounds & \quad\pounds \\
5\cdot\cancel{3}\cancel{7}\cancel{8}) & 25060(& \cdot 0466 \text{ -- } \pounds\cdot 047 \\
 & \;\;355 & \qquad\qquad\; \text{- } 11\tfrac{1}{4}\text{d} \\
 & \;\;\;\underline{33} &
\end{array}
$$

Dividing by 5 we see that the quotient begins with ·05 nearly: thus two figures only are required in quotient (we know the first figure is 0), so we keep three figures in divisor and begin with 5·37.

In most operations involving approximation it is generally sufficient to work to one place more than what is required; but in long operations (rare in commercial practice) and in some critical cases more extra figures may have to be taken.

Approximate working with two operators.

Example: Find $\dfrac{3257}{112 \times \cdot 45359}$ to two decimal places.

Move the points in the operators so that each has one unit figure:— $1\cdot 12 \times 4\cdot 5359$. One factor has been divided by 100, the other multiplied by 10, i.e., the denominator has been divided by 10: therefore divide numerator by 10: 325·7.

The fraction now is $\dfrac{325\cdot 7}{1\cdot 12 \times 4\cdot 5359}$

Make a rough outline of the work thus: 4)325(, which shows that first figure in final quotient will be two places

before the point : there will be four figures in answer, i.e., five in divisor, and as product of $1 \cdot 12 \times 4 \cdot 5359$ will have one unit figure we must work the product to four decimal places.

```
4 53590
      1 12

4 53590
 ·45359
   9072
---
5 08021)325 700(64·11(1)
         20 887
            566
             58
              8
```

Again : In 1900 in France 6,940,210 hectares of land bore 114,710,880 hectolitres of wheat. Find how many bushels per acre (to nearest whole number) this is equivalent to if a hectare $= 2 \cdot 471$ acres and a hectolitre $= 2\ 75$ bushels.

$$\text{The result is } \frac{114710880 \times 2 \cdot 75}{6940210 \times 2 \cdot 471} \text{ or } \frac{114 \cdot 710880 \times 2 \cdot 75}{6 \cdot 940210 \times 2 \cdot 471}$$

We have moved the decimal point 6 places to the left in denominator to get all operators with one unit figure, and therefore we have also moved the point six places to the left in numerator.

A rough outline gives us 12)228(, showing the result will have two figures before the point. There will thus be three figures in quotient to ensure the whole numbers being correct, and therefore we get the denominator with four figures, i.e., to two decimal places. The numerator evidently will therefore require five figures, three of which are whole numbers ; therefore we work the numerator to two decimal places.

The working is as follows :

```
6·940210      114 71088
   2 471           2·75
--------      ---------
13·880        229·422
 2·776         80·297
   486          5 736
     7        ---------
--------      315·455(18·4
17·149)       143 9
                6 8     Answer 18
```

It will be seen that the method of working such examples is to move the points so that all the operators have unit figures : from a rough outline, using the whole numbers, to estimate the number of figures in final result and hence the degree of accuracy required in the preliminary operations. In such questions it will be found best to perform multiplications before division.

EXAMPLES IV

1. Write the following correct to the number of decimal places indicated by the number in bracket : 3·45782 (3), 18·98125 (2), 60634·758 (1), 9·7163055 (3), ·0492375 (1)

2. Add together (to three decimal places) 1·47893, 2·78, 0·003695, 98·67532, 5·56789.

3. Add together (to five decimal places) $5\cdot\dot{3}$, $6\cdot\dot{7}\dot{5}$, $8\cdot\dot{4}2857\dot{1}$, $11\cdot309\dot{1}$, 0·047829.

4. Write the following as decimals of £ to three places : £5 6s. 2d., £3 13s. 5d., £169 2s. 1½d., 8s. 2¼d., £17 4s. 3d., £1 19s. 3¼d., 2s. 6d., 18s. 6d., 3s. 10½d., 7s. 9d., £49 11s. 8d., £2 1s. 7¾d.

5. Decimalise £2 3s. 1d., £15 6s. 3d., £8 17s. 2½d., £9 16s. 3½d., £14 7s. 1¼d., £1 8s. 5d., £2 16s. 3¼d., £7506 17s. 6d., £834 10s. 6d., £7 14s. 6d., £8612 3s. 8d., £99 15s 7d., £6 14s. 9d., £304 11s. 10½d., 2s. 7¼d., 6s. 8d., 14s. 2½d., 5s. 9d., 6s. 11d., 13s. 4d., £245 12s., 1½d., 2¾d., 3⅜d.

6. Express in £ s. d. correct to nearest farthing £6·112, £7·715, £3·24365, £0·27248, £3456·125, £81·9123, £1·111, £16·9375, £37·32079, £82·828, £9·146125, £·205, £·013629, £·127, £·008, £·011, £·499, £·23781, £·605833 · · · ·, £·723166 · · · ·, £·85249166 · · · ·.

7. Form the following products and quotients correct to number of places indicated by figure in brackets : 32·05725 × ·2 (3), 4·09875 × ·03 (4), 563·4728 × ·07 (4), 8812·456 × ·06 (1), ·04923 × ·05 (3), 37·24 ÷ 5 (4), 8·3075 ÷ 7 (4), 14·0235 ÷ ·2 (3), 83·30296 × 1·7 (3), 0·142857 × 1·03 (4), 8·356 × 1·025 (4), 99·7623 × 1·0101 (3), 96·27825 × 1¼ (3), 3·76925 × 2 (3), 44·23086 × 2·3 (3), 1001·2376 × 3·02 (3), 29·23 × 1·04 (4).

8. Work the following multiplications to number of places indicated by figure in brackets : 23·192765 × 2·3681 (2), ·04976 × 5·36 (3), 0·32974715 × 4·98345 (3), 56·33475 × ·2369 (2), 8·3737 · · · · × $1\cdot\dot{3}12312$ · · · · · (3), 403·652 × 71·44 · · · · · (3), 54·9173825 × ·047328 (4), ·003927 × ·00147 (6), $70\cdot7106\dot{7}$ × $1\cdot4\dot{1}4\dot{2}$ (4), 37·477353 × ·0042977 (2), 98765432·1 × 1·23456789 to nearest integer.

9. Find the following quotients correct to number of places indicated by figure in brackets: 94·73295 ÷ 3·76426 (2), 829·76 ÷ 13·715 (4), 4·262879 ÷ 923 (3), 926·527 ÷ 8·356 (3), 1253764 ÷ 90347523 (3), 41·3 ÷ 17·0569 (3), 6705·123 ÷ 9034·4725 (2), 33·795 ÷ ·7984 (2), 48353402 ÷ 5666614 to nearest integer.

10. Divide (to nearest penny) £12 13s 3d by 2240, £853 13s. 2d by 327, £10145 12s. by 9987, £137 3s 2d by 2$\frac{2}{7}$, £404 13s 6d. by 569$\frac{1}{8}$ £2 11s 5$\frac{1}{4}$d. by 27$\frac{1}{3}$, £55 17s. 9d. by 74$\frac{4}{7}$.

11. Find the rate per £ (to nearest farthing) required to raise £78946 16s on rateable value of £592345 10s.

12. In a certain year a certain trade union had 208869 members and spent £84855 on disputes. How much per head (to the nearest $\frac{1}{2}$d.) did this amount to?

13. If the assessable value of the County of London is £40142271, find what rate per £ in current coin must be levied in order to raise £2500000.

14. Decimalise £834 2s. 3d. Multiply the resulting decimal by 735. Express the answer in £ s. d.

15. Use decimals to calculate £1475 13s. 4d. $\times (1{\cdot}03)^2$.

16. Decrease £4750 by ·07 of that amount, and the result by ·07 of that result. Give the answer in £ s. d.

17. Evaluate the following, in each case to three decimal places, giving the answer in £ s. d.:

£56·73925 × 2·395 ÷ 8·3417,
£1 3s. 9d. × 2·20462125 × 25·19.
£704 17s. 3d. × 6747 ÷ 584000.

$$£\,\frac{3{\cdot}65}{1{\cdot}10231125 \times 25{\cdot}18}$$

CHAPTER V

CALCULATION OF PRICES

The first problem to be considered is the finding of the price of a number of articles, being given the price of one: e.g., to find the price of 379 articles at 2s. $9\frac{1}{4}$d. each.

(1) By compound multiplication.

s. d.
2 $9\frac{1}{4} \times 379$

£52 10 $1\frac{3}{4}$

4 | 379
94—3
3411
12 | 3505
292—1
758
20 | 1050
52—10

(2) By decimalising the money and using contracted multiplication.

$$2\text{s. } 9\tfrac{1}{4}\text{d.} = £\cdot13854166\cdots$$
$$£\cdot13854166\cdots \times 379 = £13\cdot85416\cdots \times 3\cdot79$$

£13·8541 | 6 × 3·79

41·5625
9·6979
1·2469

52·507
= £52 10s. $1\frac{3}{4}$d.

NOTE.—The working is to four places so as to ensure accuracy in the third decimal figure of the answer. In multiplying by 3, which is unit's figure, we begin with the fourth decimal figure 1, i.e., $1 \times 3 = 3$, but we carry 2 from $6 \times 3 = 18$.

The multiplication by 7 begins at the third decimal figure, 4, with 1 to carry from 7×1, i.e., $4 \times 7 + 1 = 29$. Similarly with the next line.

In using this method care must be taken that the decimalisation is carried far enough. This depends on the number of figures in the multiplier. Method No. 2 is not so convenient with large multipliers.

(3) The method known as practice is often useful. We suppose the price to be a unit, say £1 each. This gives £379 as the price ; we then take convenient parts of the unit, or of some previous part, to make up 2s. 9¼d., e.g., 2s. 6d. = $\frac{1}{8}$ of £1, 3d. = $\frac{1}{10}$ of 2s. 6d., ¼d. = $\frac{1}{12}$ of 3d. The parts to be chosen are generally fractions with unity for the numerator and a convenient divisor for denominator.

	£	s.	d.	
2s 6d. = $\frac{1}{8}$ of £1	379	0	0	at £1 each
3d. = $\frac{1}{10}$ of 2/6	47	7	6	at 2s 6d each
¼d = $\frac{1}{12}$ of /3	4	14	9	at 3d. each
	0	7	10¾	at ¼d each.
	52	10	1¾	at 2s. 9¼d each.

This method often gives awkward fractions of pence, and as most answers are only required to nearest farthing, decimals with practice will be found convenient, as each line need not be taken to more than four decimal figures :—

	£	
2s 6d. = $\frac{1}{8}$ of £1	379	at £1 each.
3d = $\frac{1}{10}$ of 2/6	47·375	
¼d. = $\frac{1}{12}$ of -/3	4 7375	
	·3948	
	52·507	= £52 10s 1¾d.

Another advantage of using decimals will be that more variety of parts is possible. We can use denominators, 20, 300, &c.

Find price of 1,125 articles at £2 0s. 6d. each.

	£	
	1125	at £1 each.
6d. = $\frac{1}{40}$ of £1	1125	at £1 each.
	28·125	at 6d. each.
	2278·125	= £2278 2s. 6d.

Find price of 204 articles at 16s. 9½d. each.

	£	
16s. = $\frac{8}{10}$ of £1	204	at £1 each.
8d. = $\frac{1}{30}$ of £1	163·2	at 16s each.
1½d = $\frac{1}{160}$ of £1	6·8	at 8d. each.
	1·275	at 1½d each.
	171·275	

=£171 5s. 6d. at 16s. 9½d. each.

Here we have taken the parts as fractions of £1 each time and divided the £204 each time. In taking $\frac{8}{10}$ of £204 we have multiplied the £204 by 8 and moved the resulting figures down one place. In taking $\frac{1}{30}$ of £204, we have divided by 3 and moved the resulting figures down one place, e.g., $20 \div 3 = 6$, but instead of placing the 6 in the tens column, we have placed it in the units column.

Similar methods are applicable when the number of articles is partly fractional.

Find price of 837¾ yards at 4s. 6½d. per yard.

	£	
4s. = $\frac{1}{5}$ of £1	837·75	at £1 each.
6d. = $\frac{1}{8}$ of 4/-	167·55	at 4s. each.
½d. = $\frac{1}{12}$ of -/6	20·9438	at 6d. each.
	1·7453	at ½d. each.
	190·239	at 4s. 6½d. each.

=£190 4s. 9½d.

or £·2270833·· × 837·75.
= £22·7083|3·· × 8·3775.

181·6666
6·8125
1·5896
·1589
·0114

£190·239
=£190 4s. 9½d.

A similar method will be found useful in calculating values of money, such as $\frac{11}{16}$, $1\frac{3}{8}$, $2\frac{2}{5}$, &c.

Find $3\frac{3}{8}$ of £27 16s. 2d.

	£	
$\frac{2}{8} = \frac{1}{4}$	27·8083	3
	83·4250	= 3 times.
$\frac{1}{8} = \frac{1}{2}$ of $\frac{2}{8}$	6·9521	= $\frac{2}{8}$,,
	3·4760	= $\frac{1}{8}$,,
	93·853	

= £93 17s. $0\frac{3}{4}$d.

or

	27 8083	3
$\frac{3}{8} = \frac{1}{8}$ of 3	83·4250	= 3 times.
	10·4281	= $\frac{3}{8}$,,
	93·853	

= £93 17s $0\frac{3}{4}$d.

The student should make himself familiar with the convenient parts. The following table will be found useful :—

Parts of £1.

s.	d.		
10	0	=	$\frac{1}{2}$
5	0	=	$\frac{1}{4}$
2	6	=	$\frac{1}{8}$
1	3	=	$\frac{1}{16}$
1	4	=	$\frac{1}{15}$
4	0	=	$\frac{1}{5}$
2	0	=	$\frac{1}{10}$
1	0	=	$\frac{1}{20}$
0	6	=	$\frac{1}{40}$
0	3		$\frac{1}{80}$
0	4	=	$\frac{1}{60}$
0	8	=	$\frac{1}{30}$
6	8	=	$\frac{1}{3}$
3	4	=	$\frac{1}{6}$
1	8	=	$\frac{1}{12}$
0	2		$\frac{1}{120}$
18	0	=	$\frac{9}{10}$
16	0		$\frac{8}{10}$
14	0	=	$\frac{7}{10}$
12	0	=	$\frac{6}{10}$
8	0	=	$\frac{4}{10}$
6	0	=	$\frac{3}{10}$
16	8	=	$\frac{10}{12}$

Parts of 10s.

s.	d.		
3	4	=	$\frac{1}{3}$
1	8	=	$\frac{1}{6}$
0	10	=	$\frac{1}{12}$
2	6	=	$\frac{1}{4}$
1	3	=	$\frac{1}{8}$
0	$7\frac{1}{2}$	=	$\frac{1}{16}$
5	0	=	$\frac{1}{2}$
4	0	=	$\frac{4}{10}$
2	0	=	$\frac{1}{5}$
1	0	=	$\frac{1}{10}$
0	6	=	$\frac{1}{20}$
0	4	=	$\frac{1}{30}$
0	3	=	$\frac{1}{40}$
0	2	=	$\frac{1}{60}$
0	$1\frac{1}{2}$	=	$\frac{1}{80}$

When the price per article differs from the unit by a convenient part the work may be shortened, as in the following example.

Price of 147 **articles** at £2 17s. 6d. each. Consider £2 17s. 6d. as £3 — 2s. 6d.

	£			
2s. 6d. = $\frac{1}{8}$ of £1	147			at £1 each.
	441			at £3 each
	18	7	6	at 2s. 6d. each.
	422	12	6	at £2 17s. 6d. each.

So also to find price of $\frac{15}{16}$ yd. at 2s. 9d. per yard.

	s.	d	
	2	9	
less $\frac{1}{16}$		$2\frac{1}{16}$	
	2	$6\frac{15}{16}$ =	2/7 nearly.

The next problem is to find the value of a compound quantity when the price of one of its denominations is given. This is known as compound practice.

Example : To find value of 20 tons 9 cwt. 3 qr. at 18s. 3d. per ton.

	£		
5 cwt = $\frac{1}{4}$ of 1 ton	0·9125	for 1 ton.	
	18·250	,,	20 tons.
4 cwt = $\frac{1}{5}$ of 1 ton	·2281	,,	5 cwt.
2 qrs. = $\frac{1}{8}$ of 4 cwt.	·1825	,,	4 cwt.
1 qr = $\frac{1}{2}$ of 2 qrs.	·0228	,,	2 qrs.
	·0114		
	18·695	for 20 tons 9 cwt. 2 qrs.	

=£18 13s. 11d.

(2) Decimalise both quantities and use contracted multiplication.

£0·9125 × 20·4875

£			
9·125 × 2·04875			
18·250	=	2	times.
·3650	=	·04	,,
730	=	·008	,,
64	=	·0007	,,
5	=	·00005	,,
18·695	=	2·04875	times
=£18 13s. 11d.			

$$\begin{array}{r|l} 4 & 3 \\ \hline 20 & 9\cdot75 \\ \hline & \cdot4875 \end{array}$$

(3) Sometimes the question may be reduced to simple practice, as in the following:—

Consider price per ton as £1, then price per cwt.=1s. and price per qr.=3d.

£	s	d	
1	1	3	Prices
20	9	3	Quantities.
20	9	9	Product

or

	£
	20·4875 by decimalising.
18s = $\frac{9}{10}$ of £1	18·4388
3d = $\frac{1}{80}$ of £1	·2561
	18·695
	£18 13s. 11d.

Example: Price of 39 ac. 2 rds. 18 sq. po. at £3 7s. 4d. per acre. Price per acre=£1, therefore price per rood=5s. and price per sq. pole=$1\frac{1}{2}$d.

£	s	d	
1	5	$1\frac{1}{2}$	Prices.
39	2	18	Quantities.
39	12	3	Product.

or

	39·6125	price at	£1	per acre.
6s. 8d. = $\frac{1}{3}$ of £1	118·8375	,,	£3	,,
8d. = $\frac{1}{10}$ of 6/8	13·2042	,,	6/8	,,
	1·3204	,,	8d.	,,
	133·362	,,	£3 7s. 4d	per acre.
	=£133 7s. $0\frac{1}{2}$d.			

Sometimes special methods may be used as :—

(1) To calculate prices per dozen from price per article we may reckon pence as shillings.

This price per doz. at $3\frac{1}{2}$d. each = 3s. 6d
,, ,, 1s. 2d. each = 14s.
,, , 3s. $4\frac{1}{2}$d. each = £2 0s. 3d.

Prices per gross can be calculated by doing this twice. At 3d. each price per doz. = 3s., and price per gross = 36s.

(2) Price of 240 articles can be calculated by reckoning price of one in pence as pounds.

Price of 240 at $2\frac{1}{2}$d. each = £2 10s.
,, 241 at 3d. ,, = £3 0s. 3d.
,, 238 at $7\frac{1}{2}$d. ,, = £7 10s. — 1s. 3d. = £7 8s. 9d.
as 238 = 240 — 2.
365 = 240 + 120 + 5.

So price of 365 at 8d. = £8 + £4 + 3s. 4d. = £12 3s. 4d., i.e., 8 at £1 + 8 at 10s. + 8 at 5d.

(3) $16 \times 15 = 240$.

So price at 1s. 3d. each may be obtained by dividing number of articles by 16, and at 1s. 4d. each by dividing by 15.

352 articles at 1s. 3d. = £22.
225 ,, 1s. 4d. = £15.
79 ,, 1s. 3d. = £4 18s. 9d.

(4) To calculate price per cwt. from price per lb. we may form a table as follows :—

	£	s	d
For 1s. a lb.,	5	12	0
,, 1d. ,		9	4
,, $\frac{1}{2}$d. ,,		4	8
,, $\frac{1}{4}$d. ,,		2	4
,, $\frac{1}{8}$d. ,,		1	2

Such a table will be useful if we have to calculate a number of such prices.

Find price of 1 cwt. if 1 lb. cost 2s. 3¼d.

Take 2 times	1/-	a lb	£11	4	0
,, 3 ,,	1d	,,	1	8	0
,, 1 ,,	¼d	,,		2	4
			£15	14	4

(5) The method of nine multiples is much used when the price remains constant for a great number of calculations. We form a table for 1, 2, 3, 4, 5, 6, 7, 8, 9 multiples.

Example: Form a table of nine multiples for a price of 5s. 2½d. each.

		£
1	=	·2604166 · ·
2	=	·5208333 · ·
3	=	·7812500
4	=	1·0416666 ·
5	=	1·3020833 · ·
6	=	1·5625000 · ·
7	=	1·8229166
8	=	2·0833333 ·
9	=	2·3437500 · ·

From the table find price of 347 articles.

			£	
Price of	300	=	78·1250	to 4 decimals
,,	40	=	10·4167	,, ,,
,,	7	=	1·8229	,, ,,
,,	347	=	90·365	
		=	£90 7s. 3¾d.	

Find price of 10,508¾ or 10,508·75.

£2604·1667
130·2083
2·0833
·1823
·0130
£2736·654

Each line is obtained from the table by moving the decimal point as required and approximating to the last figure, e.g., in price of 10,000 we moved the point four places to the right, and as the last figure was 6 and the next figure 6 we wrote 7, thus 2604·1667.

There is a similar question, viz.:—From the price of a quantity to find price per article.

Example: If price per gross is 27s. 6d., what is price per article? It is only necessary to divide by 144.

$$\begin{array}{r|rr} & s & d \\ 12 & 27 & 6 \\ \hline 12 & 2 & 3\frac{1}{2} \\ \hline \end{array}$$

$2\frac{7}{24}$d nearly $= 2\frac{1}{4}$d

Example If price per cwt. is 33s., find price per lb.

$$\begin{array}{r|r|l} & s & \\ 112 & 33 & \\ \hline & 396d. & 3d \\ & 60 & \end{array}$$

Price per article $= 3\frac{60}{112}$d. $= 3\frac{15}{28}$d. $= 3\frac{1}{2}$d. nearly.

EXAMPLES V

Find the price of (correct to nearest farthing):—

1. 343 articles at 9s. 6d each.
2. 2041 articles at £1 13s 10d. each.
3. 79 articles at £3 13s. 4d. each.
4. 815 articles at 8s. $7\frac{1}{2}$d. each.
5. 91 articles at £7 15s $4\frac{3}{4}$d. each.
6. $814\frac{2}{3}$ yards at 3s. 9d. per yard.
7. $47\frac{3}{8}$ yards at 5s. $4\frac{1}{2}$d. per yard.
8. $1023\frac{7}{16}$ yards at 11s 5d. per yard.
9. 75 lbs. at 16s. 8d. per lb
10. 654 lbs. at £2 2s. 10d. per lb.
11. 7 tons 15 cwt. 3 qrs. at 19s. 6d. per ton
12. 38 cwt. 3 qrs. 12 lbs. at £16 12s. 6d per cwt
13. 2 ac. 3 rds. 10 sq. po. at £2 5s per acre.
14. 7 miles 3 fur. 24 po. at £14 13s. 4d. per mile
15. 24 tons 3 cwt. 2 qrs. 25 lbs. at £17 11s. 8d. per ton.
16. 281 articles at £1 18s. each.
17. 87 articles at £3 17s. 6d. each.
18. 194 articles at £2 16s. 8d. each.
19. 200 articles at 8s. 4d. each.
20. 4728 articles at 2s. 11d. each.

Find the values of—

21. $\frac{9}{16}$ of £105 12s.

22. $2\frac{3}{8}$ of £21 17s.

23. $111\frac{3}{4}$ of £31 5s. 6d.

24. $\frac{5}{6}$ of £79 19s. 2d.

25. $1\frac{7}{12}$ of £209 10s.

26. Form a table of nine multiples for a price of $2\frac{1}{2}$d. each and calculate the prices of (i.) 33 articles, (ii.) 1045 articles, (iii.) 97·5 articles, (iv.) $476\frac{1}{4}$ articles, (v.) 32981 articles, each at $2\frac{1}{2}$d.

27. From the table for price of a cwt., when the price per lb. is given, calculate price of 1 cwt. when price per lb. is (i.) 3s. 3d., (ii.) $4\frac{1}{2}$d., (iii.) $2\frac{3}{8}$d., (iv.) £2 7s. $4\frac{1}{4}$d., (v.) 9s. 8d.

28. Find briefly the price per dozen when price per article is (i.) $7\frac{1}{2}$d., (ii.) $10\frac{1}{4}$d., (iii.) 2s. $2\frac{1}{4}$d., (iv.) 3s. 6d., (v.) 5s. 10d.

29. If a man puts aside $9\frac{1}{2}$d. every day, how much will he have in a year of 365 days?

30. Find price of 242 articles at 1s. 7d. each.

31. Find price of 5 cwt. 3 qrs. 11 lbs. of tea at 1s. 11d. per lb.

32. Find cost of 8 tons 11 cwt. 2 qrs. 10 lbs. at £16 6s. 8d. per ton.

33. A farmer on market day sold 90 eggs at $10\frac{1}{2}$d. per dozen, 29 lbs. of butter at 1s. $3\frac{1}{2}$d. per lb., 19 ducks at 2s. 9d. each, 28 old fowls at 1s. 9d. each, and $4\frac{1}{2}$ doz. fat chickens at 2s. 10d. each. How much did he get for all he sold?

34. How much will 15271 articles cost at £5 4s. 6d. per thousand?

35. Make out the Bill for:—15 yds. of cloth at 1s. $0\frac{3}{4}$d. per yd., 27 yds. of shirting at $6\frac{3}{4}$d. per yd., 2 pairs of blankets at 17s. 9d. a pair, $4\frac{1}{2}$ doz. reels of cotton at $2\frac{3}{4}$d. a reel, and $1\frac{1}{2}$ gross of needles at $2\frac{1}{2}$d. a dozen.

36. Find the cost of 1153 cwt. of potatoes at 4s. $5\frac{1}{2}$d. per cwt.

37. Find the price of 98 tons 17 cwt. 2 qrs. of coal at 22s. 9d. per ton.

38. Find cost of 23 yds. of silk at 5s. 4d. per yd.

39. What is the cost to the nearest penny of 17300 cubic feet of gas at 3s. 1d. per 1000 cubic feet?

40. Certain articles cost £5 12s. 6d. per ton. Find price per lb. (to nearest farthing).

41. Use the table for price per cwt. given price per lb. to calculate total cost of 2 cwt. 1 qr. 13 lbs. of sugar at $2\frac{1}{2}$d. per lb., and 1 cwt. 1 qr. 19 lbs. at $3\frac{1}{4}$d. per lb. Also find the average cost per lb.

CHAPTER VI

GENERAL PROBLEMS

In a large number of problems two quantities are connected together in such a way that a change in one causes a proportional change in the other. The change in one is given, and we are required to find the corresponding change in the other. Examples will show one common method of working such questions; this method is known as the Unitary Method.

Example: If 18 yds. of material cost 21s., what will 81 yds. of the same material cost?

We select the fact that expresses the connection between quantity of material and cost, and write it so that the cost is written last, as the answer required is a cost.

The cost of 18 yds $= 21$ sh.

$\therefore$ „ „ 1 yd. $= \frac{1}{18}$ of 21 sh.

$\therefore$ „ „ 81 yds. $= \frac{81}{18}$ of 21 sh.

$= \frac{9}{2}$ of 21 sh.

$= \frac{1}{2}$ of 189 sh.

$=$ £4 14s. 6d.

Here we change from the cost of 18 yds. to that of 1 yd. (hence the name unitary), and again to cost of 81 yds.

The actual working out of the answer $\frac{81}{18}$ of 21s. will vary; the most convenient should be chosen each time.

Example: If a journey of 60 miles takes 10 hours, how many hours would a journey of 78 miles take at the same rate?

Time for 60 miles $= 10$ hours

$\therefore$ „ 1 mile $= \frac{1}{60}$ of 10 hours.

$\therefore$ „ 78 miles $= \frac{78}{60}$ of 10 hours.

$= \frac{13}{10}$ of 10 hours.

$= 13$ hours.

Example : Find value of 3,766 lbs. at 8s. 10½d. for 14 lbs.

$$
\begin{aligned}
\text{Price of 14 lbs} &= 8\text{s. } 10\tfrac{1}{2}\text{d} \\
\therefore \quad ,, \quad 1 \text{ lb} &= \tfrac{1}{14} \text{ of } 8\text{s } 10\tfrac{1}{2}\text{d.} \\
\therefore \quad ,, \quad 3766 \text{ lbs} &= \tfrac{3766}{14} \text{ of } 8\text{s. } 10\tfrac{1}{2}\text{d} \\
&= \tfrac{1883}{7} \text{ of } 8\tfrac{7}{8} \text{ sh.} \\
&= \frac{\overset{269}{\cancel{1883}}}{\cancel{7}} \times \frac{71}{8} \text{ sh.} \\
&= 2{,}387\tfrac{3}{8} \text{ sh.} \\
&= £119 \; 7\text{s. } 4\tfrac{1}{2}\text{d.}
\end{aligned}
$$

When the student is familiar with this method, the writing out of the solution may be shortened by using what might be called the "Fractional" method.

It will have been noticed that in each case the answer is the result of multiplying by a fraction ; e.g., in the first example above,

$$\text{The cost of 81 yds.} = 21\text{s.} \times \tfrac{81}{18}.$$

The fraction consists of the two quantities whose change is given.

Example : Find the cost of 91 oranges at 7 for sixpence.

$$
\begin{aligned}
\text{Cost of 91 oranges} &= 6\text{d.} \times \frac{91}{7} \\
&= 6\text{d} \times 13 \\
&= 6\text{s. } 6\text{d.}
\end{aligned}
$$

We first decide what kind of thing is wanted ; in this case "cost." We write down the cost which is given. Then we require a multiplying fraction made up of 91 and 7 ; the only point to determine being which of the two numbers is to be on the top of the fraction. To settle this we only require to consider, will the change from 7 to 91 make the cost larger or smaller ? If larger, place the larger number on top ; if smaller, the smaller number.

The student should have practice in forming the fraction one quantity is of another. A fraction is a number, and has no denomination, such as £, lbs., yds., &c. The two quantities must be of the same denomination.

Examples: What fraction is 3s. 4d. of £1 12s. 6d. ?

i.e., 40d. of 390d.

$\therefore$ the fraction is $\frac{40}{390}$ or $\frac{4}{39}$

We might have expressed both quantities as shillings;

$$\text{the fraction} = \frac{3\frac{1}{3}}{32\frac{1}{2}} = \frac{\overset{2}{\cancel{10}}}{3} \times \frac{2}{\underset{13}{\cancel{65}}} = \frac{4}{39}$$

$$\text{Or again, as £'s} = \frac{\frac{1}{6}}{1\frac{5}{8}} = \frac{1}{\underset{3}{\cancel{6}}} \times \frac{\overset{4}{\cancel{8}}}{13} = \frac{4}{39}$$

In practice the higher denomination we choose, the smaller the figures.

It is often convenient to be able readily to express the shillings and pence as a fraction of a £. The student should make himself familiar with some of the more frequently occurring amounts, as:—2s. 6d. = £$\frac{1}{8}$, 7s. 6d. = £$\frac{3}{8}$, 12s. 6d. = £$\frac{5}{8}$, 17s. 6d. = £$\frac{7}{8}$, 6s 8d. = £$\frac{1}{3}$, 13s. 4d. = $\frac{2}{3}$, 16s. 8d. = £$\frac{5}{6}$, &c.

The following method of forming such fractions for unfamiliar amounts is useful:—

15s. 9d. = 15$\frac{3}{4}$s. = £$\frac{63}{80}$;

9d. is expressed as a fraction of a shilling; then form the improper fraction, at same time multiplying the denominator by 20.

Examples of fractional method: If 16 cwts. cost £224, what will 6 cwts. 14 lbs. cost ?

$$\text{The fraction} = \frac{6\frac{1}{8}}{16} = \frac{49}{128}$$

$$\therefore \text{cost required} = £\overset{7}{\cancel{224}} \times \frac{49}{\underset{4}{\cancel{128}}}$$

$$= £\frac{343}{4}$$

$$= £85\ 15s.$$

Example : In how many days could five men do the same work as is done by six men in 35 days ? Here a decrease in the number of men entails an increase in the time.

$$\text{Hence the fraction is } \frac{6}{5}$$

$$\therefore \text{ No. of days } = 35 \times \frac{6}{5} = \underline{42}.$$

Examples : If the tithe paid on 127 acres 3 roods 20 poles of land is £119 7s., what tithe is payable on 87 acres 1 rood 35 poles of land, assuming that the tithe per acre is the same ?

$$\text{The tithe charge will be } = \text{£}119\ 7\text{s.} \times \frac{87\frac{15}{32}}{127\frac{7}{8}}$$

$$= \text{£}\frac{\overset{7}{\cancel{2387}}}{20} \times \frac{\overset{933}{\cancel{2799}}}{\underset{4}{\cancel{32}}} \times \frac{\cancel{8}}{\underset{\cancel{341}}{\cancel{1023}}}$$

$$= \text{£}\frac{6531}{80}$$

$$= \underline{\underline{\text{£}81\ 12\text{s.}\ 9\text{d.}}}$$

Example : How long can 13 horses be kept on the same quantity of food which lasts 7 horses for 52 days ? An increase in number of horses means a decrease in time ;

$$\text{Therefore the fraction is } \tfrac{7}{13}.$$

$$\therefore \text{ Time } = 52 \text{ days} \times \tfrac{7}{13} = 28 \text{ days.}$$

The student should realise exactly what the fractional factor produces, because in many instances this process will only be part of the necessary working.

Example : A contractor engages to excavate 3,600 cubic yards in 3 days ; with 20 men he finds that 1,000 cubic yards are done in half the time. How many additional men must be put on the job to finish within the specified time ? One thousand cubic yards have been done ; 2,600 cubic yards remain to be done in same length of time. An increase of work to be done entails more men at work ; therefore the fraction will be $\frac{2600}{1000}$ or $\frac{26}{10}$. This fraction will produce the new—not the additional—number of men.

$$\therefore \text{ New number of men } = 20 \times \frac{26}{10} = 52$$

$$\therefore \text{ additional men } = 32.$$

Some applications of the "fractional" method. There are certain problems where this method is a convenient one, but in which the terms used must be known. I propose to give a short list of the terms with brief explanations. A trader who keeps his books properly should be able at any time to get out a correct statement of his affairs. When drawn out in proper form this statement is called a balance sheet. On the one side he makes a list of his liabilities or Debts that he owes; on the other a list of his Assets or the various properties with which he could pay his debts, such as Cash, Goods, Bills Receivable, Debts owing to him, &c. When his liabilities are greater than his assets, he is said to be insolvent. If he is made bankrupt his Assets must be fairly distributed amongst his creditors, and the fraction obtained by putting his Assets over his Liabilities gives the Dividend his estate can pay, generally expressed as so much a pound. If the amount of any particular creditor's claim be multiplied by this fraction, the amount that creditor will receive will be obtained.

Example: A bankrupt's liabilities amount to £6,228; his available assets are £188 2s. 9d How much can he pay in the £, and what will a creditor to whom he owes £720 receive?

The fraction for dividend $= \dfrac{188\frac{11}{80}}{6228} = \dfrac{\overset{29}{\cancel{\overset{5017}{\cancel{15051}}}}}{\underset{\underset{12}{\cancel{2076}}}{\cancel{6228}} \times 80}$

$= \dfrac{29}{960}$ or $7\frac{1}{4}$d. in the £.

The creditor for £720 will receive $£\overset{3}{\cancel{720}} \times \dfrac{29}{\underset{4}{\cancel{960}}}$

or $£\dfrac{87}{4}$

or £21 15s.

We can now consider another use of the fraction, which we can call reversing the fraction. This will be applicable in a great many questions which might be called reverse questions. The idea of this method is as follows:—Most questions are straightforward, the result being obtained by

multiplying by a certain fraction; to reverse the process or to obtain the original quantity when we know the result we may divide by the same fraction, i.e., multiply by the fraction inverted. This method will be much used later. An example will perhaps best explain the process.

Example: A bankrupt pays a dividend of 5s. 6d. in the £. One creditor received £27 18s. 3d, what was the amount of that creditor's claim?

$$\text{The dividend fraction is } \frac{5\frac{1}{2}}{20} = \frac{11}{40}$$

$$\therefore \text{ The amount of claim } = £\overset{2}{\cancel{27}}\ \overset{10}{\cancel{18}}\text{s. } \overset{9}{\cancel{3}}\text{d.} \times \frac{40}{\cancel{11}}$$

$$= £101\ 10\text{s.}$$

Example. A bankrupt pays 9s. 10d. in the £; his available assets were £1,888. Find the amount of his debts.

$$\text{The fraction for dividend } = \frac{9\frac{5}{6}}{20} = \frac{59}{120}$$

If we multiply his liabilities by $\frac{59}{120}$ we should get his assets; but we know his assets, therefore to get his liabilities we reverse the fraction.

$$\therefore \text{ Amount of Liabilities } = £\overset{32}{\cancel{1888}} \times \frac{120}{\cancel{59}} = £3840.$$

Generally in practice other points are involved in questions on bankruptcy, such as the expenses and secured creditors. These amounts must be taken off the assets to get the available assets for division amongst the ordinary creditors.

Example. A bankrupt has assets £600 and liabilities £939, of which latter £35 are preferential claims for rent, taxes, &c. Find how much in the £ the estate can receive.

After paying the £35 the available assets will be £565 and the liabilities £904.

$$\therefore \text{ The dividend fraction } = \frac{565}{904} = \frac{5}{8}$$

or 12s. 6d. in the £.

This process of reversing the fraction is convenient for a question like the following. After spending $\frac{5}{9}$ of the money in my purse, then $\frac{3}{4}$ of the remainder, and then $\frac{4}{13}$ of what was still left, I found I had 3s. $9\frac{1}{2}$d. over. How much was in the purse at first?

The fraction *after* 1st spending $= \frac{4}{9}$.

" " 2nd " $= \frac{1}{4}$ of $\frac{4}{9} = \frac{1}{9}$.

" " 3rd " $= \frac{9}{13}$ of $\frac{1}{9} = \frac{1}{13}$.

$\therefore$ Amount at first $=$ 3s. $9\frac{1}{2}$d. $\times \frac{13}{1} =$ £2 9s. $3\frac{1}{2}$d.

Taxes are charges made on property or income by Parliament. Rates are charges made by local Councils for local purposes. Income-tax is assessed on each pound of a person's income, certain allowances being recognised, the chief of which is that persons having incomes under £160 a year pay no income-tax, and persons having incomes between £160 and £400 pay tax on the amount over £160.

Rates are generally assessed on what is called the Rateable Value of the house or property, such value being fixed by persons appointed for that purpose. A man's Gross Income is his income before tax is deducted; net income after the tax is deducted.

My house is rated at £25; the rates are 5s. 6d. per £1. What do I pay in rates?

$$\text{The fraction for rate} = \frac{5\frac{1}{2}}{20} = \frac{11}{40}$$

$$\therefore \text{ I pay } £\overset{5}{\cancel{25}} \times \frac{11}{\underset{8}{\cancel{40}}} = £\frac{55}{8} = £6\ 17\text{s. }6\text{d.}$$

Inversely: If the rates are 5s. 6d. in the £, at what amount is my house rated if I pay £6 17s. 6d. for rates?

Rate fraction $= \frac{11}{40}$.

$\therefore$ Rateable value fraction $= \frac{40}{11}$.

$\therefore$ Rateable value of my house $=$ £6 17s. 6d. $\times \dfrac{40}{11}$

$$= £\frac{\overset{5}{\cancel{55}}}{\cancel{8}} \times \frac{\overset{5}{\cancel{40}}}{\cancel{11}} = £25$$

When the tax is 1s. in the £1, what does a man pay on a taxable income of £200?

$$\text{Tax fraction} = \frac{1}{20}$$

$$\therefore \text{ Tax payable} = £200 \times \frac{1}{20} = £10.$$

What is the net income of a man whose gross income is £245 when the tax is 1s. in the £1, and no tax is payable on the first £160?

The taxable income is £85.

$$\text{Tax} = £85 \times \frac{1}{20} = £4\ 5s.$$

$$\therefore \text{ Net income} = £80\ 15s. + £160$$

$$= £240\ 15s.$$

$$\text{Or we might take net income fraction } \frac{19}{20}$$

$$\therefore \text{ Net income (taxed part)} = £\overset{17}{\cancel{85}} \times \frac{19}{\underset{4}{\cancel{20}}}$$

$$= £80\ 15s.$$

$$\therefore \text{ Total net income} = £240\ 15s.$$

Unless the deduction is specially mentioned in a question, it is generally understood to be omitted altogether in working the question.

After deducting income-tax at 11d. in the £, a man's net income was £343 10s. What was his gross income?

$$\text{Net income fraction} = \frac{229}{240}$$

$$\therefore \text{ Gross income} = £343\tfrac{1}{2} \times \frac{240}{229}$$

$$= £\frac{\overset{3}{\cancel{687}}}{\cancel{2}} \times \frac{\overset{120}{\cancel{240}}}{\cancel{229}}$$

$$= £360.$$

Sometimes the required change depends on more than fraction; what is generally known as compound proportion.

Example: A colliery employs 120 men, working 8 hours a day for 5 days a week. How many men can it employ for 4 days a week working 6 hours a day, if only half as much coal is required to be raised ?

The quantity required is number of men, therefore we operate on 120 men.

The fraction due to change of days a week = $\frac{5}{4}$, because decrease of time involves increase in number of men.

The fraction due to length of working day = $\frac{8}{6}$

The fraction due to quantity of work = $\frac{1}{2}$.

$\therefore$ The combined fraction $= \frac{5}{\cancel{4}} \times \frac{\cancel{8}}{6} \times \frac{1}{\cancel{2}} = \frac{5}{6}$

$\therefore$ No. of men $= 120 \times \frac{5}{6} = 100$

Example: What is the cost of the carriage of 15 cwts. for 12 miles if 6 cwts. is carried 20 miles for 4s. ?

The fraction $= \frac{\overset{3}{\cancel{15}}}{\cancel{6}} \times \frac{\overset{\cancel{2}}{\cancel{12}}}{\underset{\underset{2}{\cancel{10}}}{\cancel{20}}} = \frac{3}{2}$

$\therefore$ The cost $= 4s \times \frac{3}{2} = 6s$

EXAMPLES VI

1. If 5 horses can plough a field in 3 days, how many horses will be required to do the work in 2 days ?

2. If a train going 10 miles an hour can cover the distance between two stations in 17 min., how long will it take at 15 miles an hour ? Give answer to nearest minute.

3. If a man's expenses are £80 for 5 months, what will they be for a year ?

4 If 5 gallons of ale cost 6s. 3d., what will 36 gallons cost at the same rate ?

5. The wages of 9 men for 5 days amount to £7 17s 6d. What is the largest number of men that can be hired at the same rate for 6 days at a cost of not more than £10 ?

6. A salesman gets 6d. for every pound's worth of goods he sells. What does he earn by selling goods worth £34,970 ?

7. If it costs a railway company £190 to carry 160 persons 400 miles, what should it cost to convey half that number 250 miles? Give the cost to nearest £.

8. If 3 men can reap 8 acres in 5 days, how many days can 30 men reap 192 acres?

9. A bankrupt pays 11s 4d. in the £. How much does he pay out of a debt of £37?

10. If 7 yds. of cloth cost 30s., what (to nearest 1d.) will be the cost of 10 yds. 2 ft. of the same cloth?

11. A train travels 225 miles in 6 hours. How long will it take to travel 135 miles at three-quarters of its former rate?

12. My income last year was £750. If I paid an income-tax of 1s. in the £, and spent on the average 32s. 6d. per day, how much did I save?

13. If the carriage of 3 cwts. for a certain distance be 16s., what will the carriage of 12 tons be for the same distance?

14. If it cost £13 15s. to feed 50 men for a week, what will it cost to feed 60 men for the same time?

15. A householder paid £19 2s. 6d. in rates and £60 in rent. Express the rates as a fraction, in its lowest terms, of the rent.

16. To how much does a tax of 1s. 2d. in the £ on £729 15s. amount?

17. If 50 gallons of water flow through a pipe in 6 min., how many minutes, to nearest minute, will it take for 1,744 gallons to flow through?

18. A contractor finds that a certain piece of work can be done by 16 men in 6 days of 8 hours each. How many men must be set to work at it if he wishes to have it finished within four days, the men working 9 hours a day?

19. After paying an income-tax of 10d. in the £, a man's net income was £690. If the income-tax had been a shilling in the £, what would his net income have been?

20. A garrison of 3,500 men, with provisions for 60 days at the rate of 27 oz. per day, is reinforced by 1,000 men, and cannot be relieved for 90 days. How many ounces per day can be allowed to each man so that the provisions may just last without further reduction of rations?

21. A contractor undertakes to construct a railway in 2 years; he employs 540 men, and completes half the work in $\frac{5}{8}$ of the time. How many additional men must be put on to finish it by the specified date?

22. A bankrupt has assets £600, and liabilities £930, of which latter amount £210 are preferential claims for rent, taxes, &c., and must be paid in full. How much in the £ ought his creditors to receive?

23. If a bankrupt's assets amount to £1,820 6s. 3d., and he can pay 9s. 8½d. in the £, what is the amount of his debts?

24. A railway company issues return tickets at a single fare and a quarter; what would the return fare be if the single fare is 2s. 2d., and what would the single fare be when the return is 12s. 6d.?

25. A bankrupt has as his assets debts due to him as follows:—£470 9s. 3d., £283 4s. 6d., £70 6s. 8d., £110 18s. 3d.; when realised the first turns out good, and for the others he receives respectively 6, 7, and 10 shillings in the £. His liabilities amount to £4,600. How much can he pay in the £?

26. The Poor Rate being 3s. 4d. in the £, find the rateable value of a property which pays a rate of £416 13s. 4d.

27. If a bankrupt pays 13s. 4d. in the £, what will a creditor for £2,179 19s. 3d. receive?

28. A man has £236 a year to spend after deducting an income-tax of 4d. in the £ on his gross income, and putting away one-fifth of what remains. Find his gross income.

29. A person pays an annual premium of £8 6s. for life insurance. His income-tax amounts to £6 12s. 6d. when the rate is 1s. in the £. No tax is paid on the first £160 of his income nor on his insurance premium. Find his gross income.

30. If 25 gas burners, which are lighted 5 hours a day for 20 days, consume £2 2s. 6d. worth of gas, how many burners can be lighted 4 hours a day for 30 days at a cost of £7 13s.?

CHAPTER VII

PERCENTAGES

The term "per cent." is one that is often met with in business. A percentage of a quantity simply means a certain fraction of that quantity, the denominator of the fraction being 100. So that 3%, $2\frac{1}{2}$%, 25%—read as 3 per cent., $2\frac{1}{2}$ per cent., 25 per cent.—are equivalent to

$$\frac{3}{100}, \quad \frac{2\frac{1}{2}}{100}, \quad \frac{25}{100} \text{ respectively.}$$

Questions involving "per cent." can be worked by forming the proper fraction and operating with it.

The various styles of questions that involve per cent. will now be illustrated by examples, and in the succeeding chapters we shall discuss the applications to special types of questions.

(1) To calculate a percentage of a given quantity. Form the fraction corresponding to the given rate per cent., reduce it to its lowest terms, and multiply the given quantity by the fraction.

Examples. Find 10% of £32.

$$10\% = \frac{10}{100} = \frac{1}{10}$$

$$\therefore \text{ required percentage is } £32 \times \frac{1}{10} = £3\ 4s.$$

Find $33\frac{1}{3}$% of £68.

$$\text{The fraction} = \frac{33\frac{1}{3}}{100} = \frac{1}{3}$$

$$\text{percentage} = £68 \times \tfrac{1}{3} = £22\ 13s.\ 4d.$$

Sometimes it is better to form the decimal fraction; e.g., 3% = ·03, $2\frac{1}{4}$% = ·02$\frac{1}{4}$ or = ·0225.

Examples : Find $2\frac{1}{4}\%$ of £1,568 7s. 6d.

$$
\begin{array}{rcl}
 & 2\frac{1}{4}\% = \cdot 02\frac{1}{4}. & \\
\therefore & \text{£}1{,}568\cdot 375 \times \cdot 02\tfrac{1}{4} & \\
 & 31\cdot 3675 = \cdot 02 & \text{times} \\
 & 3\cdot 9209 = \cdot 00\tfrac{1}{4} & ,, \\
 & \overline{35\cdot 288} & \\
\end{array}
$$

= £35 5s. $9\frac{1}{4}$d. (to nearest farthing).

Here we have decimalised the money to three places. To find 2%, or ·02, of the quantity we have multiplied by 2, moving the figures two places to the right. Also as we need only work to 4 places, we began by multiplying the 7; thus, $2 \times 7 + 1$ (to carry from 5×2) $= 15$, i.e., 5 and carry 1. To get the $\cdot 00\frac{1}{4}$ times, or $\frac{1}{4}\%$, we might divide by 4, moving the figures two places to the right, or we might have taken $\frac{1}{8}$ of ·02 times.

The student should choose the fraction which seems the easier; generally the decimal method is advisable. We might also vary the decimal method by moving the point two places to the left in the quantity, thus giving 1% of it at once; then multiply by $2\frac{1}{4}$.

Example : Find $2\frac{1}{4}\%$ of £315 2s. 10d.

$$
\begin{array}{ll}
\text{£} & \\
3\cdot 15{,}141 & 1\% \\
\overline{6\cdot 3{,}028} & 2\% \\
\cdot 7878 & \frac{1}{4}\% \\
\overline{7\cdot 090} & \\
\end{array}
$$

= £7 1s. $9\frac{3}{4}$d.

For mental or brief working the student should make himself familiar with certain rates per cent. and their equivalent fractions; e.g., 5% is $\frac{1}{20}$, or, in the case of money, 1s. in the £. $2\frac{1}{2}\%$ is $\frac{1}{40}$, or, in the case of money, 6d. in the £; $33\frac{1}{3}\%$ is $\frac{1}{3}$, $66\frac{2}{3}\%$ is $\frac{2}{3}$, $12\frac{1}{2}\%$ is $\frac{1}{8}$, $8\frac{1}{3}\%$ is $\frac{1}{12}$, 25% is $\frac{1}{4}$, 50% is $\frac{1}{2}$, $3\frac{1}{3}\%$ is $\frac{1}{30}$, &c.

Example : 5% of £25 7s. 6d. = £1 5s. $4\frac{1}{2}$d., since it is = $25\frac{3}{8}$s.

In the case of money, the following method is often used if the sum be £'s only. Multiply the number of £'s by twice the rate, cut off the units digit; the rest will be shillings, the figure cut off is reckoned as pence and fifths of a penny.

Example : Find $1\frac{1}{2}\%$ of £95.

$$\begin{array}{l} 95 \\ \underline{3} \ (=1\tfrac{1}{2} \times 2) \\ 28{,}5 = £1\ 8s.\ 6d.\ (= 28s. + 5d. \text{ and } 5 \times \tfrac{1}{5}d.). \end{array}$$

$$\begin{aligned} \text{Reason :—}£95 \times \frac{1\frac{1}{2}}{100} &= £95 \times \frac{3}{200} \\ &= 95s. \times \frac{3}{10}. \\ &= 28{\cdot}5s. \\ &= 28s. + \tfrac{5}{10} \text{ of } 12d. \\ &= 28s. + 5 \times \tfrac{6}{5}d. \\ &= £1\ 8s.\ 6d. \end{aligned}$$

(2) The reverse process :—Given that a certain quantity is a given percentage of some unknown quantity, to find the unknown quantity.

Calculate the fraction as before, and multiply by the fraction inverted. This will, of course, give the original quantity.

Of what number is 29, 2% ?

$$2\% = \frac{2}{100} = \frac{1}{50}.$$

$$\therefore \text{ The no.} = 29 \times \frac{50}{1} = 1{,}450.$$

As multiplication by $\frac{1}{50}$ would give 29 as 2% of the required number, multiplication of 29 by 50 must give the required number. Compare this with similar questions in Chapter VI.

Example : In a school 270 pupils are learning arithmetic, and these form 75% of the total number of pupils. What is the total number ?

$$75\% = \frac{75}{100} = \frac{3}{4}.$$

$$\therefore \text{ Total number} = 270 \times \frac{4}{3} = 360.$$

(3) The next type of question is to increase or decrease a given quantity by a given percentage. There are two methods open to us. First method—the more natural method —is to calculate the increase or decrease, and add to or subtract from, the original quantity.

Example : The cost of provisions for a charitable institution last year was £3,451 17s. 6d. This year the cost has risen $7\frac{1}{2}\%$. What will be the amount this year ?

$$7\tfrac{1}{2}\% = \frac{15}{200} = \frac{3}{40}.$$

$$£3{,}451\ 17s\ 6d \times \frac{3}{40} = \frac{£10{,}355\ 12s.\ 6d.}{40}$$

$$= £258\ 17s\ 10d.\ \text{(to nearest 1d.)}.$$

$\therefore$ Cost this year = £3,710 15s. 4d.

Or, by decimals,

£	
3,451·875	
241·6313	7%
17·2593	$\frac{1}{2}\%$
3710·766	

= £3,710 15s. 4d

Here by keeping the decimal points underneath each other we get the adding with less work.

Second method is to form the fraction including the increase or decrease per cent., i.e , the fraction which will produce the new quantity ; e.g., in the above we form the fraction =

$$\frac{107\frac{1}{2}}{100} = \frac{215}{200} = \frac{43}{40}$$

or = $1{\cdot}07\frac{1}{2}$. This latter form gives the same as in the second working above.

Example : If we take 25% from £3 15s., what sum is left ?

First method :—

$$25\% = \tfrac{1}{4}.$$

$$£3\ 15s. \times \tfrac{1}{4} = £0\ 18s.\ 9d.$$

$\therefore$ Amount left is £2 16s. 3d

Second method :—

$$\text{Fraction} = \frac{75}{100}\ (75 = 100 - 25)$$

$$= \tfrac{3}{4}$$

$$\therefore \text{amount left} = £3\ 15s. \times \tfrac{3}{4} = £11\ 5s. \times \tfrac{1}{4}$$

$$= \underline{£2\ 16s.\ 3d.}$$

(4) The reverse of question 3 is simple if the second method is known, as an inversion of the fraction will give the required operator.

Example: If £2 16s. 3d. is the net amount after deduction of 25%, what was the gross amount? $\frac{75}{100}$ or $\frac{3}{4}$, is the fraction to find net amount when gross amount is known. ∴ $\frac{4}{3}$ is fraction to produce gross amount when net amount is known.

∴ Net amount £2 16s. 3d. × $\frac{4}{3}$

= £0 18s. 9d. × 4

= £3 15s.

Students often go wrong in this type of question through not clearly recognising that the percentage is calculated on the original quantity, in this case the unknown quantity.

Example: Between 1891 and 1901 the population of a certain town was said to have increased by 21%. In 1901 it was 123,297. What was it in 1891?

Many students make the following mistake. They calculate 21% of 123,297 and subtract, but what is meant by an increase of 21% is that the population at 1891 increased by 21% of what it then was, and the result is 123,297; i.e., the increase is 21% of the 1891 population, but they take it as 21% of 123,297. This point requires consideration, and if the method of reversing the fraction is understood the correct method is easily seen.

$$\text{The fraction for 21\% increase} = \frac{121}{100}$$

$$\therefore \text{ fraction to go backwards is } \frac{100}{121}$$

$$\therefore \text{ population in 1891} = 123{,}297 \times \frac{100}{121}$$

$$= 101{,}898$$

The population, of course, must be expressed in whole numbers.

Verification:—

101,898·	1 time
20,379·6	·2 time
1,018·98	·01 time
123,297·	

(5) The remaining problem is one of very common occurrence, viz., from given particulars to calculate the rate per cent. Care must be taken to see which quantity forms the basis of the rate. The rate we have seen is a fraction; therefore we calculate the fraction and then change the form of the fraction to one with 100 as denominator.

Examples.

(1) What percentage is equivalent to the fraction $\frac{1}{20}$?

$$\text{The fraction} = \frac{1}{20} = \frac{5}{100} = 5\%.$$

(2) What percentage is equivalent to the fraction $\frac{31}{39}$?

$$\text{The fraction} = \frac{31}{39} = \frac{31 \times 100}{39 \times 100} = \frac{3100\%}{39}$$

$$= 79\tfrac{19}{39}\%$$

$$\text{or} = 79{\cdot}5\% \text{ to one place of decimals}$$

(3) If 9 increases to 10, what is the rate of increase per cent.?

$$\text{Fraction} = \frac{1}{9} = \frac{100}{9}\% = 11\tfrac{1}{9}\%.$$

Here the actual increase is 1; this gives numerator of fraction; the quantity that increases is 9, this gives the denominator, and so the rate is $\frac{1}{9}$.

(4) If 10 decreases to 9, what is the rate of decrease per cent.?

$$\text{Fraction} = \frac{1}{10} = 10\%.$$

(5) During 1903 the imports of raw cotton into the United Kingdom were as follows:—

		£
From	Egypt	9,643,295
,,	United States	32,345,746
,,	Brazil	928,750
,,	British East Indies	1,584,388
,,	Other Countries	333,143

What percentage of the total import (to the nearest whole number) came from the United States?

The total import = £44,835,322

$$\therefore \text{ The fraction} = \frac{32,345,746}{44,835,322} \times 100\%$$

$$= \frac{323 \cdot 45746}{4 \cdot 4835322}\%$$

$$= 72\%.$$

$$4 \cdot 4835)323 \cdot 46(72$$
$$9\ 61$$
$$65$$

See Chap. IV. for contracted division

(6) A bicyclist increases his rate in miles per hour by $12\frac{1}{2}\%$; by how much per cent. does he decrease his time for a mile?

$$\text{The speed fraction} = \frac{112\frac{1}{2}}{100}$$

$$\therefore \text{ the time fraction} = \frac{100}{112\frac{1}{2}}$$

$$\therefore \text{ decrease fraction} = \frac{12\frac{1}{2}}{112\frac{1}{2}}$$

$$= \frac{\cancel{25}}{\underset{9}{\cancel{225}}} \times 100\%$$

$$= 11\tfrac{1}{9}\%.$$

(7) A manufacturer finds that by using a new kind of coal which is 10% more in cost than what he was using previously, he gets same work from 10% less amount of coal. Which is the more economical, and by what rate per cent.?

$$\text{The price fraction} = \frac{110}{100}$$

$$\text{the quantity fraction} = \frac{90}{100}$$

$$\therefore \text{ the compound fraction} = \frac{110}{100} \times \frac{90}{100} = \frac{99}{100}$$

showing a decrease of $\frac{1}{100}$ or 1%.

EXAMPLES VII

1. Write down, in their lowest terms, the fractions equivalent to $3\frac{1}{3}\%$, 24%, 15%, 100%, 35%, 1%, $\frac{1}{2}\%$, $\frac{1}{8}\%$, $4\frac{1}{8}\%$, $1\frac{1}{2}\%$, $33\frac{1}{3}\%$

2. Express the following fractions as rates per cent. :—$\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{7}$, $\frac{1}{6}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{13}{16}$, $\frac{77}{80}$, ·02, ·21, ·225, ·0125, ·04375, ·0275.

3. Calculate the following percentages.—5% of £110, 5% of £57 13s. 4d., 5% of 17s. 6d., 5% of £1,024 9s., $2\frac{1}{2}\%$ of £10, $2\frac{1}{2}\%$ of £13 15s., $2\frac{1}{2}\%$ of £16 4s., 2% of 1,050, 3% of 119, $4\frac{1}{2}\%$ of 147,586, $3\frac{1}{3}\%$ of £59, $4\frac{3}{8}\%$ of £1,031 5s., $22\frac{1}{2}\%$ of £67, 75% of £1,016 6s. 6d., 20% of £66 13s. 4d.

4. Increase the following quantities by the given rates per cent :—£60 by 3%, £795 10s. by $1\frac{1}{2}\%$, 33,491 by 27%, 40,362 by $3\frac{1}{8}\%$, 882 by 11%, £6,541 by $2\frac{1}{2}\%$, £33,465 10s. 9d. by $7\frac{1}{2}\%$; decrease the following by given rates per cent :—36,789 by 1%, 4,857 by 3%, 999 by 10%, £891 13s. 4d. by $12\frac{1}{2}\%$, £7,074 by 7%, £3,636 16s. by $33\frac{1}{3}\%$, £9,567 by $1\frac{1}{8}\%$, £374,915 by $11\frac{1}{4}\%$, £1,325 6s. 10d. by 60%.

5. Of what number is 267, 3% ?

6. Find the gross amount if when $8\frac{1}{3}\%$ is deducted there is £12 17s. left.

7. To half a pint of spirit, water is added to make up a quart ; what percentage of the mixture is spirit ?

8. A man has an income of £325, and spends £314. Express the ratio of his expenses to his income as a decimal to two places.

9. A water company charges $7\frac{1}{2}\%$ on the house rent. How much is this on a rent of £50 ?

10. A trading company pays £4 on a share of £200. What percentage is this ?

11. The national expenditure for the year 1901–2 included the following payments to Local Taxation accounts :—England, £8,296,747; Scotland, £1,125,830 ; Ireland, £1,145,403. What percentages (to nearest whole number) were received respectively by England, Scotland, and Ireland ?

12. A block of ice, originally weighing 2 kilogrammes, has partly melted away, till only 56 per cent. of it remains. Give the weight of what remains in grams.

13. The average cost per week of food consumed by an agricultural labourer's family in England during 1902 was as follows :—Bread and flour, 3s. 5d.; meat, 4s. 2d.; other articles of food, 5s. $11\frac{1}{2}$d. Express the amounts spent (1) upon bread and flour, (2) upon meat, as percentages of the whole amount spent upon food.

14. In a village 70 per cent. of the grown people are married. If there are 2,460 grown people how many married are there ?

15. A man earns 32s. a week, and pays 2s 6d. a week to a Friendly Society. What percentage is this of his wages ? (Answer to nearest whole number)

16 The following table shows the quantity of tea imported by Great Britain in the two years 1878 and 1900. Find (to the nearest whole number) what percentage came from China in each year .—

—	In 1878.	In 1900
	lbs	lbs.
Holland	3,145,000	6,594,000
China ..	165,656,000	21,316,000
India .	35,423,000	153,581,000
Ceylon .	1,000	114,180,000
Other Countries	647,000	2,929,000

Also find by what percentages the import from China has decreased, and that from Ceylon increased

17. A man devotes 15 per cent. of his income towards insurances, and on this he pays no income tax After paying 1s. in the £ on the rest, he finds he has £1,250 left. Find his gross income to the nearest pound.

18. The income of the London County Council from all sources amounted in 1899 to £3,533,560. A rate of 1s. 1½d. in the £ on the assessable value of the County of London produced 55 per cent. of this income. Find, to the nearest pound, how much is raised by a rate of 1d. in the £.

19. For the year ending 31st December, 1901, the gross receipts and working expenses of certain railways were as follows . —

Railway.	Gross Receipts.	Working Expenses.
L. & Y.	£5,503,997 ..	£3,460,201
L. & N.W.	13,915,925 ..	8,829,760
G.W.	11,388,399 ..	7,215,948

Express in each case the working expenses as a percentage (correct to one place of decimals) of the gross receipts.

20. If a man receives 12½% of £554 9s. 4d., 4% of £459 7s. 6d., and 13% of £585, how much does he receive altogether ?

21. Through failure of crops, &c., an annual income of £2,000 has rapidly diminished. In one year it fell 4%, in the next year 15% of the remainder, and in the next year 25% of what then remained. What was its amount at the close of the third year?

22. What is the freight on 480 bales of cotton weighing 4 cwt. 4 lb. each at $\frac{3}{4}$d. per lb. and 5% additional.

23. Between two census takings the population of a town increased by $21\frac{1}{4}$%; at the second census the population was 234,801. What was it at the first census?

24. When the duty on a commodity is reduced by 40%, the consumption is increased by 60%. By how much per cent. is the revenue from it increased or decreased? With what increase of consumption would the revenue remain unchanged?

25. A workman in the country saves £10 a year. He moves to the town, where he earns 10% more, spends 5% more, and saves £5 a year more. What was his original income?

CHAPTER VIII

COMMISSION, BROKERAGE, DISCOUNT FOR CASH, INSURANCE

In this chapter we propose to discuss certain simple applications of percentages, and the methods of last chapter are directly applicable. The questions to be discussed will be such as do not contain the element of time.

A good deal of business is done by means of agents. An agent may be defined as a person appointed to transact business in the name and for the account of another. His remuneration is usually in the form of an allowance on the amount of the business transacted, which is fixed by special agreement, or in accordance with the usage of the trade. This remuneration is generally called the agents' commission, and in most cases is a percentage of the amount of the transaction.

The agent is sometimes styled a Broker, and his commission is known as brokerage. For example, we have Stockbrokers, Bill-brokers, &c.

Example: We employ an agent to sell for us on $3\frac{1}{4}\%$ commission. His sales amount to £895 16s. 8d. What do we pay the agent?

Working:—	£8·95833	1%
	26·8750	3%
	2·2395	$\frac{1}{4}\%$
	29·115	

Commission = £29 2s. 4d. (to nearest 1d.)

An agent charges $2\frac{1}{4}$ per cent. commission for disposing of a property. If the expenses of the sale amounted to

£25 10s., and the agent, after deducting this and his commission, hands over £707 12s. 6d., for what was the property sold ?

$$£707\ 12s.\ 6d. + £25\ 10s. = £733\ 2s.\ 6d$$

$$\text{The fraction for amount less commission} = \frac{97\frac{3}{4}}{100} = \frac{391}{400}$$

$$\therefore \text{Total price} = £733\tfrac{1}{8} \times \frac{400}{391} \text{ (inverting fraction).}$$

$$£\frac{\overset{15}{\cancel{5865}}}{\cancel{8}} \times \frac{\overset{50}{\cancel{400}}}{\cancel{391}}$$

$$= \underline{£750}$$

The manager of a department is paid a salary of £250 and a commission of $2\frac{1}{2}$ per cent. on the sales in his department. If this raises his salary to £365 in a certain year, what is the aggregate of the sales in his department that year ?

His commission = £115

$$\text{The commission fraction} = \frac{2\frac{1}{2}}{100} = \frac{1}{40}$$

$$\therefore \text{ Amount of sales} = £115 \times 40 = £4{,}600.$$

Another term frequently met with is Discount. The simplest meaning is an amount deducted from a bill for cash payment or for prompt payment. For example, we often find printed on invoices something like the following : Terms, $2\frac{1}{2}\%$ discount monthly account. If the amount of sales to us amount to (say) £11 9s. 4d., then if we pay at beginning of next month we are required to pay £11 3s. 7d., as $2\frac{1}{2}\%$ on £11 9s. 4d. (say £11 10s.) is 5s. 6d.

Generally, as in above example, the discount is calculated at a percentage rate. These rates vary in different businesses.

Another form of discount—Trade Discount—is that of a deduction from the invoice price of the goods sold. The manufacturer or wholesale dealer issues to his customers,

the retail merchants, a price list with the full prices attached. He arranges to supply these at a certain rate of discount. For example, retail booksellers very often buy from the publishers at a discount of 25% from the published prices of the books. Sometimes the two forms of discount are combined, as in the following example. I arrange with an agent in London to supply me with books on the following terms: — 25% discount from published prices, and a further 5% for cash within an agreed upon term. Invoices arrive similar to following: —

London, Feb. 3/05

G. H. DOUGLAS, Esq.,

Bradford

BOUGHT OF BROWN, JONES & CO.,

School Booksellers,

ALDERSGATE, E.C

		Subject to Discount			Net		
		£	s	d	£	s	d
13 Smith's Conics ..	3/6	2	5	6			
27 Balladen und Romanzen, net	2/6				3	7	6
1 Pitman's Reporter .		0	2	0			
		2	7	6			
25%		0	11	10			
		1	15	8			
5%			1	8	1	14	0
				£	5	1	6

25% of £2 7s. 6d. = 11/10. This is the trade discount.

5% of £1 15s. 8d. = 1/8. This is the cash discount.

Another example of trade discount :—

INVOICE.

LIVERPOOL, *7th July*, 1905.

Mr. E. CHARLES, Chemist, Leeds.

BOUGHT OF JAS. ALLISON & CO. LTD

Terms : Net 1 mo.

				£	s.	d.
20%	$\frac{1}{6}$	doz. Cameras, $\frac{1}{4}$ pt. 00.	150/-	1	5	0
20%	1	,, Xylonite Trays, $\frac{1}{4}$ pl. asstd. cols	4/6	0	4	6
25%	4	,, bxs. Peerless Plates, $\frac{1}{4}$ in. spec. rap.	12/-	2	8	0
Net	2	Cks Hypo. lbs				
		No. 1.... 1 1 12 16				
		No. 2.... 1 1 3 14				
		2 2 15 30				
		0 1 2				
		2 1 13 @ per cwt	10/6	1	4	10
				5	2	4
		Less 20% on £1 9 6 .. 5s. 10d.				
		,, 25% on £2 8 0 .. 12s. 0d.		0	17	10
				4	4	6
		1 Hamper		0	5	0
				4	9	6

The articles are sold from a printed price list under varying rates of discount.

Fire insurance is a contract wherein one party—the Insurance Company—undertakes to pay to the other the pecuniary loss by fire which may happen to specified property during a particular period, in consideration of an immediate fixed payment. This payment is called a premium, and is calculated as a certain percentage of the amount for which the insurance is made. There are other forms of insurance, as Life Assurances, Employers' Liability Insurance, &c.

Examples: Find the premium to insure a life for £300 at £2 7s. 9d. per cent.

$$\text{The premium fraction} = \frac{2\frac{31}{80}}{100}$$

$$\therefore \text{ The premium} = £300 \times \frac{2\frac{31}{80}}{100}$$

$$= £2\ 7s.\ 9d. \times 3.$$

$$= £7\ 3s.\ 3d.$$

I wish to insure goods valued at £365. The rate charged is 3s. 6d. per cent. Find amount of premium.

$$\text{The premium} = £365 \times \frac{\frac{7}{40}}{100}$$

$$= £\frac{511}{800} = £{\cdot}639$$

$$= \underline{12s\ 10d.}$$

I pay £8 6s. 6d. as insurance premium on my life, the rate charged being £2 10s. per cent. What sum am I insured for ?

$$\text{Premium fraction} = \frac{2\frac{1}{2}}{100} = \frac{1}{40}$$

$$\therefore \text{ Amount} = £8\ 6s.\ 6d. \times 40$$

$$= \underline{£333}$$

I value my house at £5,850. The rate of fire insurance is £2 10s. per cent. I wish to insure the house, so that in case of fire I shall be indemnified for the house and also for the premium paid. What premium must I pay ?

Evidently I must insure the house for an amount greater than £5,850: the fraction for which increased value

$$= \frac{100}{97\frac{1}{2}} = \frac{200}{195}$$

$$\text{Insured value} = £\overset{30}{\cancel{5850}} \times \frac{200}{\cancel{195}} = £6,000$$

$$\text{The premium} = £150.$$

It is assumed that if the house is burned down the insurance company will pay £6,000, so that I receive £5,850, the value of the house, and £150, the premium paid. But in practice this would be very doubtful, as the company only undertake to make good the loss, and they do not necessarily accept our valuation of the property.

EXAMPLES VIII

1. Calculate commission on the following amounts:—5% on £625, $2\frac{1}{2}$% on £3,400, $7\frac{1}{2}$% on £660 12s., $\frac{1}{8}$% on £7,525, $\frac{1}{4}$% on £6,844 12s. 9d., $3\frac{1}{2}$% on £1,007 16s. 2d., 20% on £79 6s. 3d., $1\frac{1}{4}$% on £299, $\frac{5}{8}$% on £8,254 15s

2. Deduct the following discounts:—$22\frac{1}{2}$% from £486 19s. 9d., $17\frac{1}{2}$% from £317 16s 3d., 35% from £210, $1\frac{1}{2}$% from £1,736 15s., $9\frac{1}{16}$% from £963 7s. 9d., $5\frac{7}{8}$% from £1,000

3. Find the premiums for insuring:—£1,000 at 3s. 6d per cent., £875 at £2 1s. 8d. per cent., £23,415 at £1 5s. per cent., £275 16s at 2s. 8d. per cent.

4. An agent collects rents, the gross value of which is £391 12s. On this he charges a commission of $4\frac{1}{2}$%. What is the net amount the landlord receives?

5. The owner of a ship's cargo pays a premium, which at the rate of 7% amounts to £514 6s. What is the cargo valued at?

6. A rate collector receives 3% for his services. How much has he collected if he receives £125?

7. The gross rental of an estate is £1,250. Income-tax at 1s. in the £ is deducted, and the cost of collection is 3% on the remainder. Find the net rental.

8. Find cash price of goods sold as follows:—List price, £22 10s. with discount 5%; £11 7s. 9d. with discount 4%, £38 16s. 2d. with discount $22\frac{1}{2}$%.

9. What rate per cent. of discount is equivalent to 2d. in the 1s.?

10. What percentage is allowed to an agent who receives £65 13s. on a sale of £2,273 10s.?

11. Find the premium necessary to effect an assurance of £550 on the life of a person aged 30, if the rate for that age is £2 11s. per cent.

12. What sum should be insured, to cover loss and premium, when the value of the goods is £485, the rate 3%?

13. A bookseller, in rendering his account, makes a deduction of 3d. in the 1s. from the published price. For prompt payment he makes a further deduction of $2\frac{1}{2}$% off the account as rendered. What fraction of the published price does he then receive?

14. A merchant owns a ship which was valued at £11,772. Find for what sum he must insure the ship so that in the event of total loss he may recover not only the value of the ship, but also the premium of 4% of the sum insured and the fee of $\frac{1}{2}$ per cent. of the sum insured to the agent for recovering the money.

15. Make out an invoice for the following sale of books, terms 25% discount off published prices (except in case of books priced net), and a further 5% for prompt cash :—6 Allcroft's History of Greece @ 4s. 6d., 2 Pitman's Manual @ 1s. 6d., 2 Matriculation Chemistry @ 5s. 6d., 1 Adgie's Book-keeping, Pt. I, at 1s. 6d., 3 French Reader @ 9d., and 2 Greek Primer at 3s. 6d. net.

16. Make out invoice for following goods :—2 quires P O P @ 18s. per quire, discount $33\frac{1}{3}$ per cent.; 3 doz $\frac{1}{4}$-plate printing frames @ 7s. 6d. per doz, discount 25% ; 1 doz 15 gr. tubes chloride of gold @ 21s. a doz, net ; 2 cks. carbonate of soda @ 24s. cwt., net ;

	cwts.	qrs.	lbs.	Tare in lbs.
No. 1 . . .	1	2	10	15
No. 2 . . .	1	1	9	18

Hamper 4s.

CHAPTER IX

PROFIT AND LOSS

One of the main inducements to a person to take up a business is to make a profit by selling articles at higher prices than what they cost him. Unfortunately it is not always possible to do this, and if a tradesman is forced to sell an article at a lower price than it cost him, he is said to make a loss. The profit or the loss is measured by the difference between the cost price and the selling price. Generally profits and losses are reckoned at so much per cent. on *cost* price. This is to be assumed in all questions except where it is specially known to be otherwise. Very often in actual business profits and losses are reckoned as percentages of the takings, i.e., the prices at which goods are sold. It is outside our province to discuss which method is better; so far as the arithmetic of such questions is considered, the same principles are applicable if it be clearly understood which basis of reckoning is meant. So I repeat that in ordinary questions Cost Price is taken as the basis, unless when other basis is specially mentioned. The types of questions are similar to those in the general chapter on percentages.

I.—To calculate actual profit or loss made on a transaction or series of transactions. Find cost price, including all expenses, and selling price of the particular goods referred to in question; the difference gives profit when selling price is greater than cost price, and loss when selling price is less than cost price.

Example: (1) How much do I gain by buying 240 yds. of cloth at 3s. 9d. a yard, and selling it at 5s 7d. a yard?

240 yds. at 3s 9d a yard - £45
240 yds at 5s. 7d. a yard £67

. Gain is £22

or, Gain on 1 yd is 1s. 10d

∴ Gain on 240 yds is £22

We can multiply price by 240 by reckoning pence in price as £1.

(2) I import 5,000 cigars at 5s. 4d. per 100. The duty is equivalent to 6s. per 100, and I reckoned my expenses at 6d. per 100. I sell the lot at 17s. per 100. What is my gain ?

The total cost of 100 = 5/4 + 6/- + 6d = 11/10

The receipts per 100 = 17/

∴ Profit on 100 = 5/2

∴ Total profit = 5/2 × 50

= £12 18s 4d

II.—To reckon profit or loss per cent. We find cost price and selling price of an article, or of the same quantity of a commodity; then write down the fraction that the gain is of the *cost* price; this fraction is then expressed as a rate per cent.

Examples

(1) I buy an article for 4s and sell it for 5s

Rate of profit = $\frac{1}{4}$ = 25%.

(2) Articles bought at 3d. per dozen are sold at 6d. per score; find profit per cent.

Cost price of 1 doz = 3d

Cost price of 1 score = 3d × $\frac{20}{12}$ = 5d

Selling price of 1 score = 6d.

∴ Rate of profit = $\frac{1}{5}$ = 20%

We have to be careful that we find the two prices of the same quantity, e.g., in above we have found the two prices per score. Note also that the denominator is the *cost* price.

(3) I buy from a price list at a discount of 60%, and sell from the same price list at a discount of 35%. What is gain per cent. ?

I buy at 40 and sell at 65.

∴ Rate of profit = $\frac{25}{40} \times 100\% = 62\frac{1}{2}\%$.

(4) I sell 7 apples at the same price as I gave for 10. What is rate of profit per cent. ?

$$\text{The fraction for quantity at same price} = \frac{7}{10}$$

$$\therefore \text{ The fraction for price of same quantity} = \frac{10}{7}$$

$$\therefore \text{ Rate of profit} = \frac{3}{7} \times 100\% = 42\tfrac{6}{7}\%.$$

(5) I buy a horse for £35 and sell it to a friend for £45. Later he is forced to sell it, and only gets £35. What is my gain per cent. and my friend's loss per cent. ?

$$\text{My gain} = \frac{10}{35} \times 100 = 28\tfrac{4}{7}\%.$$

$$\text{My friend's loss} = \frac{10}{45} \times 100 = 22\tfrac{2}{9}\%.$$

Note the denominators in the two cases.

(6) I buy a piece of cloth measuring 74 yds. at 4s. 6d. per yd. It is invoiced to me at 37 yds. equivalent to 36 yds. I sell it at 5s. 6d. per yd. Find gain per cent. ?

		£	s.	d
74 yds. @ 4/6	=	16	13	0
Less $\frac{1}{37}$		0	9	0
Cost price	=	16	4	0

Selling price = 5/6 × 74 = £21 5s.

Profit = £5 1s.

$$\therefore \text{ Rate of profit} = \frac{5\frac{1}{20}}{16\frac{1}{5}} = \frac{101}{\cancel{20}} \times \frac{5}{81} \times \overset{5}{\cancel{100}}\,\%$$

$$= 31\tfrac{14}{81}\%$$

III.—Given cost price and rate of profit or loss, to find selling price.

Two methods are open to us :—

(1) Calculate profit on cost price and add.

(2) Form fraction to include profit, and so obtain a fraction to give selling price.

Examples :

(1) A horse cost £75 ; at what price must it be sold to make $5\frac{1}{4}\%$ profit ?

$5\frac{1}{4}\%$ of £75 = £3 18s. 9d.

∴ Selling price = £78 18s 9d.

or, Fraction for selling price $= \frac{105\frac{1}{4}}{100} = \frac{421}{400}$

$$\begin{array}{r} 75\cdot \\ \hline 3\cdot75 \\ \cdot1875 \\ \hline 3\cdot9375 \end{array}$$

$$\therefore \text{ Selling price} = £\frac{\overset{3}{\cancel{75}} \times 421}{\underset{16}{\cancel{400}}}$$

= £78·937

= £78 18s 9d

The first method will be found sufficient for simple questions ; in complex questions the second will be better.

(2) Certain articles cost 36s. 7d. per dozen ; mark them per article to gain 16%.

$$\text{Fraction} = \frac{116}{100} \times \frac{1}{12} = \frac{116}{1200} = \frac{1\cdot16}{12}$$

$$\begin{array}{r} \text{d.} \\ 439\cdot \\ 43\cdot9 \\ 26\cdot34 \\ \hline 12)\overline{509\cdot24} \\ \hline 42\cdot44 \end{array}$$

Selling price per article = 3s. $6\frac{1}{2}$d.

$$\begin{array}{r} \text{or}\quad £1\cdot82916 \\ \cdot1829 \\ \cdot1097 \\ \hline 12)\overline{2\cdot122} \\ \hline \cdot177 = 3/6\tfrac{1}{2}\text{d.} \end{array}$$

IV. - Questions involving discount for cash. Here there are really three prices—Cost Price, Marked (or List) Price to include a rate of profit, and Cash Price after discount has been deducted from List Price.

Examples :

(1) A tradesman has articles costing him 3s. each. He marks them to show a gain of 20%, but he allows a discount

off the marked price of $2\frac{1}{2}\%$. What is his actual rate of profit per cent. ?

$$\begin{tikzcd} \text{Cost price} \arrow[r, "\frac{120}{100}"] & \text{List price} \arrow[dl] \\ \text{Cash price} \ \frac{97\frac{1}{2}}{100} & \end{tikzcd}$$

The Fraction from Cost Price to List Price $= \dfrac{120}{100} = \dfrac{6}{5}$

The Fraction from List Price to Cash Price $= \dfrac{97\frac{1}{2}}{100} = \dfrac{195}{200} = \dfrac{39}{40}$

$\therefore$ Fraction from Cost Price to Cash Price $= \dfrac{\cancel{6}^{\,3}}{5} \times \dfrac{39}{\cancel{40}_{\,20}} = \dfrac{117}{100}$

$\therefore$ Fraction for profit $= \dfrac{17}{100} = 17\%$

If the student will examine the diagram and the directions of the arrows, the question may appear simpler.

(2) In the former question how much per cent. above cost price should the tradesman have marked the goods so as to have 20% profit *after* allowing a discount of $2\frac{1}{2}\%$ from list price ?

$$\begin{tikzcd} \text{Cost Price} \ \frac{120}{100} \arrow[dr] & & \text{List Price} \ \frac{97\frac{1}{2}}{100} \arrow[dl] \\ & \text{Cash Price.} & \end{tikzcd}$$

The Fraction from Cost Price to Cash Price $= \dfrac{120}{100} = \dfrac{6}{5}$

The Fraction from List Price to Cash Price $= \dfrac{97\frac{1}{2}}{100} = \dfrac{39}{40}$

$\therefore$ Fraction from Cash Price to List Price $= \dfrac{40}{39}$

$\therefore$ Fraction from Cost Price to List Price $= \dfrac{\cancel{6}^{\,2}}{\cancel{5}} \times \dfrac{\cancel{40}^{\,8}}{\cancel{39}_{\,13}}$

$= \dfrac{16}{13}$

$\therefore$ Fraction for increase $= \dfrac{3}{13} \times 100\% = 23\frac{1}{13}\%$

A consideration of the diagram will show that one fraction is direct and the other inverse, as shown by the arrows.

The rule is often given, use fraction $\frac{100 + \text{profit}}{100 - \text{discount}}$

By way of verification consider an article costing 13s.

An increase of 20% makes it $15\frac{3}{5}$ sh
After taking $2\frac{1}{2}\%$ discount from $15\frac{3}{5}$ sh.
The selling price $= 15\frac{21}{100}$ sh.

$$\therefore \text{ Profit rate} = \frac{2\frac{21}{100}}{13} \times 100\% = \frac{221}{13}\% = 17\%.$$

Again, $13/- + 23\frac{1}{13}\%$ of $13/- = 16/-$.
$16/-$ less $2\frac{1}{2}\%$ of $16/- = 15\frac{3}{5}$ sh.

$$\therefore \text{ Rate of profit} = \frac{2\frac{3}{5}}{13} \times 100\% = 20\%.$$

V.—Given selling price and rate of profit per cent., to find cost price.

Form fraction as in second method of III., and use it inverted.

The fraction as formed gives the selling price if we multiply the cost price; hence if we multiply selling price by same fraction reversed we get the cost price. The student must be reminded that the rate of profit is based on the *cost* price, which in this case is not known, but is the quantity to be found.

Examples :

(1) If we sell at a profit of 15%, what was cost of an article we sell at 4s. $9\frac{1}{2}$d. ?

$$\text{The Fraction} = \frac{115}{100} = \frac{23}{20}$$

$$\therefore \text{ Cost Price} = 4/9\tfrac{1}{2} \times \frac{20}{23} = \frac{\overset{5d.}{\cancel{115}}}{\cancel{2}} \times \frac{\overset{10}{\cancel{20}}}{\cancel{23}} = 4/2.$$

(2) A man loses $12\frac{1}{2}\%$ by selling goods at 5s. 10d. a lb. What did they cost him ? How much must he raise the price to gain 10% ?

$$\text{Fraction} = \frac{87\frac{1}{2}}{100} = \frac{175}{200} = \frac{7}{8}$$

$$\therefore \text{ Cost price} = 5/10 \times \frac{8}{7} = 6/8.$$

$$\text{Again, new fraction} = \frac{110}{100} = \frac{11}{10}$$

$$\therefore \text{ New selling price} = 6/8 \times \frac{11}{10} = 7/4.$$

(3) By selling goods at 3s. a lb., a merchant gains 20%. At what price should they be sold to gain 10% ?

$$\text{Fraction for first selling price} = \frac{120}{100} = \frac{6}{5}$$

$$\therefore \text{ Cost price} = 3/- \times \frac{5}{6} = 2/6.$$

$$\text{Fraction for second selling price} = \frac{110}{100} = \frac{11}{10}$$

$$\therefore \text{ Cost price} = 2/6 \times \frac{11}{10} = 2/9.$$

Or,

$$\text{Fraction for second selling price} = \frac{100}{120} \times \frac{110}{100} = \frac{11}{12}$$

$$\therefore \text{ Second selling price} = 3/- \times \frac{11}{12} = 2/9.$$

(4) If the manufacturer makes a profit of 20%, the wholesale dealer a profit of 25%, and the retail shopkeeper a profit of 40%, what was the cost of the manufacture of an article bought at a shop for 17s. 6d. ?

To connect shop price to cost price we have fraction

$$= \frac{\overset{\overset{3}{\cancel{6}}}{\cancel{120}}}{\underset{\cancel{5}}{\cancel{100}}} \times \frac{\overset{\cancel{5}}{\cancel{125}}}{\underset{\underset{\cancel{2}}{\cancel{4}}}{\cancel{100}}} \times \frac{\overset{7}{\cancel{140}}}{\cancel{100}} = \frac{21}{10}$$

$$\therefore \text{ Cost price} = 17/6 \times \frac{10}{21} = \frac{\overset{5}{\cancel{35}}}{\cancel{2}} \text{ sh.} \times \frac{\overset{5}{\cancel{10}}}{\underset{3}{\cancel{21}}} = 8/4.$$

VI.—Given actual profit or loss, and the rate per cent., to find the cost price.

Find fraction for profit or loss and use it inverted.

Examples.

(1) An article is sold for 9d. profit, equivalent to 10% profit. What was the cost price ?

$$\text{Profit fraction} = \frac{10}{100} = \frac{1}{10}$$

$$\therefore \text{ Cost price} = 9\text{d.} \times 10 = 7/6.$$

(2) The loss on a consignment was £111, or at a rate of 7½% of the original value; find that value.

$$\text{Loss fraction} \quad \frac{7\frac{1}{2}}{100} = \frac{15}{200} = \frac{3}{40}$$

$$\therefore \text{ Original value} = £111 \times \frac{40}{3} = £1{,}480.$$

EXAMPLES IX

1. Find the rate of profit or loss per cent., being given:—

Cost Price.	Selling Price.
10/	12/6
£1 2s.	£1
4½d.	7½d.
10½d	7⅓d.
£15 12s. 6d.	£20
£20	£15 12s. 6d.

2. Find the selling price of goods, being given:—

The Cost Price	And Rate of Profit.
5/	15%.
7½d	25%.
£1 11s	8⅓%.
3/1	33⅓%.
18/-	40%.

3. Find the selling price of goods, being given:—

The Cost Price.	And Rate of Loss.
5/	15%.
7½d.	25%.
£1 11s	8⅓%.
3/1	33⅓%.
18/-	40%.

4. Find the cost price, being given:—

Selling price	And Rate of Profit.
£10	40%.
9/9	17%.
£5 8s	8%.
11/11	17½%.
9/-	25%.

5. Find the cost price, being given:—

Selling Price.	And Rate of Loss.
£23 15s 4d.	8%.
17/6	20%.
7½d.	50%.
£4 11s.	13⅓%.
£2 19s. 7d.	35%.

6. I buy coals at the colliery at 6s. 3d per colliery ton (i.e., 21 cwts.), and sell at 18s 4d. per ordinary ton, the cost of carriage from the colliery being 6s. 9d. a ton. What is my profit per cent. ?

7. Eggs are bought at 12 for 10d. and sold at 10 for a shilling. What is the profit per cent ?

8. I buy goods at £4 13s. 4d. per cwt How shall I sell them per lb to give at least 30% profit ?

9 If the profit on an article is 2s. 6d , equivalent to $16\frac{2}{3}$%, find cost price and selling price.

10. A butcher has been selling meat at 10d. a lb. and making a profit of 20% on what it cost him. If he now has to pay 8% more, and raises his price 1d. per lb , what percentage of profit does he make ?

11. A sells to B an article at a profit of 20% B sells it to C at a profit of 5% If C pays 70s , what did it cost A (to the nearest shilling) ?

12. A tradesman sold 5 cwt. 3 qr. 11 lb. of tea at 1s. 11d. per lb., and made a profit of £19 2s 1d Express the profit as a percentage of the cost price of the tea.

13. Goods are sold for £440 12s. 6d. at a loss of 6%. Find the cost price. What gain per cent. would have been made if the goods had been sold for £500 ?

14. A man buys 300 eggs at 15 for 1s., and sells them at 10d. per dozen. Find (1) his actual (2) his percentage gain or loss.

15. A firm allows its customers 25% discount on list prices, and a further discount of 5% on this reduced price if the account is settled within 3 months. A customer having received these successive reductions in his account, pays them £18 3s. $4\frac{1}{2}$d. What was the list price of the goods he had of them ?

16. A manufacturer estimates that he can make a certain patented article for £17 per gross. Find to the nearest penny the retail price for each article, which must be fixed so that, after paying the patentee a royalty of 10% on the retail price and giving the retail tradesman a discount of 40% on the retail price, the manufacturer may make a profit of 25% on the cost price. If the retail price were fixed at 6s., what would be the actual amount of profit made by the manufacturer on each article ?

17. Find price per lb. to gain 10% if goods are bought at £1 5s. 9d. per cwt.

18. A tradesman's expenses are 12% of his receipts, and he allows his customers 5% abatement for cash How much above cost price must he mark his goods to make a profit of 10% on his capital ?

19. A is a manufacturer, B a wholesale dealer, C a retail dealer, A sells to B, making a profit of 10% on the cost of production; B sells to C, making a profit of 15% on A's selling price, and C makes a profit of 20% on his own selling price. What was cost of production of goods retailed at £202 8s.

20. A tradesman buys at a discount of 10% and sells at an advance of 20% on nominal cost price. Supposing he loses $\frac{1}{2}$ an oz. a lb., find his percentage of profit.

21. If a bookseller is able to buy books at a discount of 25% from published prices, and a further discount of 5% for cash, and is able to sell at a discount of 2d in the shilling, what is his rate of profit per cent.?

22. If 100 articles of a given kind can be made in a week out of £40 worth of raw materials, cost of labour being £10 per week and fixed charges for rent, &c., £350 a year, find (*a*) cost price per article, (*b*) invoice price in order that a profit of 30% on prime cost may be realised, the following allowances being necessary:—10% commission to agents on money received for sale, 12% for bad debts; (*c*) total profits (to nearest £) for a year.

23. A book costs a publisher 4s 6d for printing. At what price should he sell the book in order that he may make a profit of 30% after paying the author 10% on the selling price?

24. If a tobacconist makes a profit of 28% by selling tobacco at 4d per oz., what will be his percentage of profit on the same tobacco if he sells it at $3\frac{1}{2}$d. per oz. after the duty has been reduced from 3s 2d. to 2s. 8d per lb?

25. What must be sale price of an article whose cost price is £1, so that a profit of 20% may be made after giving the customer a discount of 10%?

26. A bookseller sells books at a discount of 25% on the published price. He buys them at a discount of $33\frac{1}{3}$% on the published price, and gets 13 to the dozen. Find the bookseller's profit.

27. Tea bought at 1s. 8d per lb was sold at a profit of 5%; the entire profit on it was £1 8s. How many pounds of it were bought and sold?

28. Find the rate of profit per cent. made by a grocer who, by selling eggs at 20 for a shilling, makes a profit of $1\frac{1}{2}$d on every dozen sold.

29. A tradesman marks his goods at prices which would yield 25% profit on his outlay, but he allows his customers 10% discount. If his receipts are £88 2s. 6d., find what his profits are.

30. A bicycle manufacturer prints in his lists a price which would give him a profit of 60% on the cost of manufacture, and then allows for cash a discount of 22%. What profit per cent. does he actually make?

31. If a man gain 4% by selling articles at 10s. 10d. each, what price must he charge for 100 that he may gain £100 ?

32. A tradesman bought a quantity of tea and sold it at 2s. per lb., thereby gaining $33\frac{1}{3}$%. If his total gain was £14, what weight of tea did he buy and sell ?

33. What percentage profit on cost is $33\frac{1}{3}$% on selling price ?

34. A dealer purchases goods and sells them at a profit of $12\frac{1}{2}$%. How much does he receive for every £ of his outlay ?

35. If fish are bought 16 for a shilling, what must be their selling price to make $33\frac{1}{3}$% profit ?

36. A bicycle agent allows 25% discount on his advertised prices, and then makes a profit of 20% on his outlay. What is the advertised price of the machine on which he gains £3 ?

37. A tradesman sells articles at 14s 7d. per 100, and makes 3d. profit on every dozen sold What rate per cent. is this on his buying price ?

38. One bookseller allows his customers 2d. in the shilling discount, and in addition 5% on the remaining price; another simply allows 20% on the published price. Find which terms are the best for the customer by considering the case when the published price of the books is £6.

39. By selling potatoes at 16s. $11\frac{1}{2}$d. per sack, a dealer lost $7\frac{1}{2}$%. At what price should he have sold in order to have gained $12\frac{1}{2}$% ?

40. A man sells 110 articles for the cost price of 120; find his rate per cent. of profit.

41 A man sold goods for £753 more than he gave for them, and cleared 15%. What sum did he buy them for ?

42. A merchant marks his goods at 10% above cost price. What is the utmost workable discount he can allow so that he may not lose ?

43. A trader buys 100 articles for £12 13s How much must he charge for each so that 8% of the money received may be profit ?

CHAPTER X

SIMPLE INTEREST: CALCULATION

When one person borrows money from another, or uses another person's money, he has to pay for that use. The payment for the use of money is generally called interest. For example, a banker's business largely consists of borrowing money from one set of people and of lending to another set. To the former the banker pays interest, and from the latter he receives interest. Again, when an account is not paid till some time after it is due, the debtor is generally charged interest. The universal practice is to reckon the interest as a percentage. As compared with the previous questions on per cent., another factor, that of time, is introduced. If two persons borrowed similar amounts of money at equal rates of interest, it is evident that the one who kept the money for the longer time should pay the more interest. So we find the rate of interest generally quoted at so much per cent. per annum, e.g., $3\frac{1}{2}\%$ per an. If "per annum" is not stated it is implied.

THE CALCULATION OF SIMPLE INTEREST.

Examples :—

1. Find simple interest on £475 for $3\frac{1}{3}$ years at $7\frac{1}{2}\%$ per annum.

$$\text{The interest fraction} = \frac{\overset{1}{\cancel{10}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{\overset{1}{\cancel{5}}}{\cancel{15}}}{\underset{\underset{4}{\cancel{20}}}{\cancel{200}}}$$

$$\therefore \text{Interest} = £475 \times \tfrac{1}{4} = \underline{\underline{£118\ 15s.}}$$

The sum of money borrowed is called the principal. The student will find it convenient to work out the fraction before multiplying the principal.

It is often useful to express the interest fraction as a per cent. for the period, as in the above example :—

$$\therefore \frac{10}{3} \times \frac{15}{2}\% = 25\%$$

$$\begin{array}{rl} £4 \cdot 75 = & 1\% \\ \hline 95 \cdot 0 = & 20\% \\ 23 \cdot 75 = & 5\% \\ \hline £118 \cdot 75 & \\ = £118 \ 15s & \end{array}$$

This latter form is very useful when results do not come out exactly and interest is only calculated to the nearest penny. In practical questions we rarely find periods longer than a year, and below will be found suggestions for calculating interest for months or days.

Interest for months.

Examples :—

1. Find interest on £361 10s. 8d. for 5 mos. at 3%.

$$\text{Interest fraction} = \frac{5}{12} \times 3\% = 1\tfrac{1}{4}\%$$

$$\begin{array}{r|l} £3 \cdot 6153 & 3\ldots = 1\% \\ \cdot 9038 & = \tfrac{1}{4}\% \\ \hline 4 \cdot 519 & \\ = £4 \ 10 \ 5 & \end{array}$$

2. What shall I pay for interest if I borrow £450 for 8 mos. at 5%?

$$\text{Interest fraction} = \frac{2}{3} \times 5\% = 3\tfrac{1}{3}\%$$

$$\begin{array}{|rl} £4 \cdot 5 = & 1\% \\ \hline 13 \cdot 5 = & 3\% \\ 1 \cdot 5 = & \tfrac{1}{3}\% \\ \hline £15 \cdot 0 & \end{array}$$

Or 5% for one year is equivalent to 1s. in the £, i.e., £22 10s. and $\tfrac{2}{3}$ of £22 10s. = £15.

3. Find interest on £239 17s. 4d. for 10 mos. at $4\frac{1}{2}\%$.

$$\text{Interest fraction} = \frac{\overset{5}{\cancel{10}}}{\underset{4}{\cancel{12}}} \times \frac{\overset{3}{\cancel{9}}}{\cancel{2}}\% = 3\tfrac{3}{4}\% = (4 - \tfrac{1}{4})\%$$

£2·3986|6	=	1%
9·5946	=	4%
·5996		$\frac{1}{4}$%
8·995		

= £8 19 11

Interest for days.

Examples :—

1. Find interest on £751 8s. 2d. for 61 days at 2%.

$$\text{Interest fraction} = \frac{61}{365} \times \frac{2}{100}$$

$$= \frac{61 \times 4}{73000}$$ (We have doubled numerator and denominator.)

$$\therefore \text{ Interest} = £751{\cdot}408 \times \frac{244}{73000}$$

$$= £7{\cdot}51408 \times \frac{2{\cdot}44}{7{\cdot}3}$$ (Point moved four places to left in numerator and denominator so as to give operators with unit figure, see Chapter IV.).

$= £2{\cdot}511$

$= £2\ 10\ 3$

WORKING.

```
      7·51408
      2·44
     --------
     15·02816
      3·00563
       ·30056
     --------
7·3)18·33435(2·511
      3·73
        84
        113
         40
```

The method known as the "Third, Tenth, Tenth" is often used. We shall first give an example of its working and afterwards an explanation.

Working.

£·0075	1408
1·5028	200 times.
3006	40 "
300	4 "
1·8334	
·6111	$\frac{1}{3}$
611	$\frac{1}{10}$
61	$\frac{1}{10}$
2·512	
= £2 10 3	

Rule.

(*a*) Decimalise the principal and move point five places to left.

(*b*) Multiply by product of number of days and *twice* the rate (244).

(*c*) Add $\frac{1}{3}$ of this product, and $\frac{1}{10}$ of this $\frac{1}{3}$, and $\frac{1}{10}$ of $\frac{1}{10}$ of this $\frac{1}{3}$.

In multiplying by 200 we begin multiplication at 6th decimal figure, moving the result two places up.

Explanation of special method for division by 73000 :—

$$\frac{1}{73000} = \cdot00001369.. = \cdot0000137 \text{ nearly}$$

$$= \frac{1\cdot37}{100000}$$

The denominator shows the reason of step (*a*) in rule.

Again, 1·37 may be written as $1 + \cdot33\frac{1}{3} + \cdot03\frac{1}{3} + \cdot00\frac{1}{3}$ or $1 + \frac{1}{3} + \frac{1}{10}$ of $\frac{1}{3} + \frac{1}{10}$ of $(\frac{1}{10}$ of $\frac{1}{3})$.

We have taken $\frac{1}{73000} = \frac{1\cdot37}{100000}$

i.e., $100000 = 100010$

or $10000 = 10001$

73 × 1·37
21 9
5·11
100.01

This shows that for £10 of interest as thus calculated, the error is $\frac{1}{4}$d. too much.

Example :—

2. Find interest on £12,465 15s. 9d. for 85 days at $3\frac{1}{2}\%$.

$85 \times 7 = 595$

£	
·1246	5787..
62·3280	= 500 times
11·2191	90 ,,
·6233	5 ,,
74·1713	
24·7237	$\frac{1}{3}$
2·4724	$\frac{1}{10}$
·2472	$\frac{1}{10}$
101 615	

= £101 12 $3\frac{3}{4}$, less 10 farthings.

= £101 12 1 to nearest penny.

3. Find simple interest on £725 deposited in a bank from March 13th to August 2nd of the same year at $2\frac{3}{4}\%$.

From March 13th to March 31st..	= 18	days.
April	= 30	,,
May	= 31	,,
June	= 30	,,
July	= 31	,,
August	= 2	,,
Total	= 142	,,

$142 \times 5\frac{1}{2}$

710
 71

781

£	
·0072	5
5·075	= 700 times
·5800	= 80 ,,
73	= 1 ,,
5·6623	
1·8874	$\frac{1}{3}$
·1887	$\frac{1}{10}$
189	$\frac{1}{10}$
7·757	

= £7 15 2

The number of days between two given dates may also be found from a "Table of Days." Such a table gives for each day a number which is the number of days from the beginning of the year; the required number of days is found by subtracting the number for the first day from that for the second date, e.g., for March 13th the table gives 72, for August 2nd, 214, the difference being 142.

If the year be leap year, we must add one day to each after February 28th.

TABLE OF DAYS

	Jan	Feb	March	April	May	June	July	August	Sept	Oct	Nov	Dec
1	1	32	60	91	121	152	182	213	244	274	305	335
2	2	33	61	92	122	153	183	214	245	275	306	336
3	3	34	62	93	123	154	184	215	246	276	307	337
4	4	35	63	94	124	155	185	216	247	277	308	338
5	5	36	64	95	125	156	186	217	248	278	309	339
6	6	37	65	96	126	157	187	218	249	279	310	340
7	7	38	66	97	127	158	188	219	250	280	311	341
8	8	39	67	98	128	159	189	220	251	281	312	342
9	9	40	68	99	129	160	190	221	252	282	313	343
10	10	41	69	100	130	161	191	222	253	283	314	344
11	11	42	70	101	131	162	192	223	254	284	315	345
12	12	43	71	102	132	163	193	224	255	285	316	346
13	13	44	72	103	133	164	194	225	256	286	317	347
14	14	45	73	104	134	165	195	226	257	287	318	348
15	15	46	74	105	135	166	196	227	258	288	319	349
16	16	47	75	106	136	167	197	228	259	289	320	350
17	17	48	76	107	137	168	198	229	260	290	321	351
18	18	49	77	108	138	169	199	230	261	291	322	352
19	19	50	78	109	139	170	200	231	262	292	323	353
20	20	51	79	110	140	171	201	232	263	293	324	354
21	21	52	80	111	141	172	202	233	264	294	325	355
22	22	53	81	112	142	173	203	234	265	295	326	356
23	23	54	82	113	143	174	204	235	266	296	327	357
24	24	55	83	114	144	175	205	236	267	297	328	358
25	25	56	84	115	145	176	206	237	268	298	329	359
26	26	57	85	116	146	177	207	238	269	299	330	360
27	27	58	86	117	147	178	208	239	270	300	331	361
28	28	59	87	118	148	179	209	240	271	301	332	362
29	29	—	88	119	149	180	210	241	272	302	333	363
30	30	—	89	120	150	181	211	242	273	303	334	364
31	31	—	90	—	151	—	212	243	—	304	—	365

Example .—

4. What is the interest on £1,425 from July 17th to September 4th at $2\frac{1}{8}\%$?

July 17th is 198, September 4th 247 ; No. of days, 49.

$$\begin{array}{r} 49 \times 4\frac{1}{4} \\ \hline 196 \\ 12\frac{1}{4} \\ \hline 208\frac{1}{4} \end{array}$$

$$\begin{array}{r|l} \pounds & \\ \cdot 0142|5 & \\ \hline 2{\cdot}850 & = 200 \text{ times} \\ \cdot 1140 & = 8 \quad ,, \\ \cdot 0035 & = \tfrac{1}{4} \quad ,, \\ \hline 2{\cdot}9675 & \\ \cdot 9891 & \tfrac{1}{3} \\ 989 & \tfrac{1}{10} \\ 99 & \tfrac{1}{10} \\ \hline 4{\cdot}065 & \end{array}$$

$= \pounds 4 \ 1\text{s} \ 1\text{d}$

If the interest be added to the principal, the result is called the amount.

Example :—

5. Find amount of £605 lent for 3 months at $3\frac{1}{2}\%$.

Interest fraction $= \frac{3}{12} \times \frac{7}{2}\% = \frac{7}{8}\% = (1 - \frac{1}{8})\%$.

$$\begin{array}{rl} \pounds 6{\cdot}05 & = 1\% \\ \cdot 75625 & = \tfrac{1}{8}\% \\ \hline 5{\cdot}293 & \end{array}$$

$= \pounds 5 \ 5\text{s.} \ 11\text{d.}$

$\therefore$ Amount $= \pounds 610 \ 5\text{s.} \ 11\text{d.}$

Or, Interest fraction $=\frac{1}{4}\times\frac{7}{200}=\frac{7}{800}$

$\therefore$ Amount fraction $=\frac{807}{800}$

$\therefore$ Amount $=£605\times\frac{807}{800}$

$=£\cdot75625\times807$

$=$**£610 5s. 11d.**

$$\begin{array}{r|l} \cdot7562 & 5 \\ \hline 605\cdot000 & \\ 5\cdot2938 & \\ \hline 610\cdot294 & \end{array}$$

6. On January 1st, 1905, my balance at the bank was £351 7s. 8d. I paid in between that date and June 30th amounts as follows:—January 27th, £55; February 4th, £102 10s.; February 26th, £93; May 20th, £79 12s. 3d. I drew out £30 on February 12th and £87 10s. on June 11th. The bank allows 2% per annum on the daily balance. What amount is to my credit at June 30th?

The Balances are:—

Jan.	1	..	£351	7	8
,,	27	.	406	7	8
Feb.	4	..	508	17	8
,,	12	..	478	17	8
,,	26	..	571	17	8
May	20	..	651	9	11
June	11	..	563	19	11

The corresponding days are 27 (since the money was there on December 31st, 1904), 8, 14, 83, 22, 19; see Table of Days.

Working:—

$$\begin{array}{r|l} £ & \\ \cdot0035 & 1383 \\ \hline \cdot3514 & \\ 281 & \\ \hline \cdot3795 & \\ \cdot1265 & \\ 127 & \\ 13 & \\ \hline \cdot520 & = 10/5 \end{array}$$

$27\times4=108$

£·0040	6383	$8 \times 4 = 32$	£·0050	86	$8 \times 4 = 32$
·1219			·1526		
81			102		
·1300			·1628		
433			543		
43			54		
4			5		
·178 = 3/7			·223 = 4/6		

£·0047	86383	$14 \times 4 = 56$	£ ·0057	163	$83 \times 4 = 332$
·2393			1·7149		
287			·1714		
			114		
·2680			1 8977		
893			·6325		
89			633		
9			63		
·367 = 7/4			2·600 = £2 12s.		

£·0065	1495	$22 \times 4 = 88$	£·0056	3995	$19 \times 4 = 76$
·5211			·3947		
521			338		
·5732			·4285		
1911			·1428		
191			143		
19			14		
·785 = 15/9			·587 = 11/9		

	£	s.	d.
∴ Interest =	0	10	5
	0	3	7
	0	4	6
	0	7	4
	2	12	0
	0	15	9
	0	11	9
	5	5	4
Add last Balance	563	19	11
	£569	**5**	**3** = **Balance at June 30th, 1905.**

Or the figuring may be reduced by forming the products of the balances by the respective numbers of days and calculating interest on sum of products.

		Balance.			Days	Products.	
Jan.	1	£351	7	8	27	9477	
,,	27	406	7	8	8	3248	
Feb.	4	508	17	8	8	4072	
,,	12	478	17	8	14	6706	
,,	26	571	17	8	83	47476	
May	20	651	9	11	22	14322	
June	11	563	19	11	19	10716	
,,	30	5	5	3		·9601|7	× 4
Balance		£569	5	2		3·8407	
						1·2802	$\frac{1}{3}$
						·1280	$\frac{1}{10}$
						128	$\frac{1}{10}$
						5·261	

In practice it is customary to neglect the shillings and pence when they are under 10s. and to reckon the pounds as one more when shillings are over 10s., as in third balance above we have multiplied 509 by 8 to obtain product 4072.

Some banks reckon interest on the minimum monthly balance.

Example :—

7. The balance at December 31st, 1904, was £20 11s. 10d. The account for January, 1905, was as follows :—

Jan.	1.	Balance	£20	11	10
,,	5.	Drawing	2	10	0
			18	1	10
,,	6.	Deposit	51	5	1
			69	6	11
		Drawing	15	16	6
			53	10	5
,,	11.	Drawing	7	1	3
			46	9	2

Interest is allowed at 3% on minimum monthly balance, i.e., 3% for 1 mo. on £18 1s. 10d., or 11d.

EXAMPLES X

1. Calculate simple interest on (i.) £750 for 2 years at $1\frac{1}{2}\%$, (ii.) £18 15s. for 5 years at 5%, (iii.) £1,439 10s. 6d. for $1\frac{1}{2}$ years at 7%, (iv.) £365 for 2 years 8 mos. at $2\frac{1}{4}\%$, (v.) £693 11s. for 15 years at 2%.

2. Calculate simple interest on (i.) £60 for 1 year at $3\frac{1}{2}\%$, (ii.) £405 for 6 mos. at 2%, (iii.) £850 10s. for 3 mos at $1\frac{1}{8}\%$, (iv.) £1,091 16s. 3d. for 4 mos. at $2\frac{1}{2}\%$ (v.) £1,206 15s. for 5 mos. at $3\frac{3}{4}\%$, (vi.) £987 12s. 6d for 10 mos. at $2\frac{1}{4}\%$, (vii.) £83 11s for 5 mos. at $2\frac{3}{8}\%$, (viii.) £965 for 3 mos. at $4\frac{1}{2}\%$, (ix.) £375 for 7 mos. at 4%, (x.) £12,500 for 9 mos. at $3\frac{1}{3}\%$.

3. Calculate simple interest on (i.) £637 for 29 days at 3%, (ii.) £347 10s. for 101 days at $2\frac{1}{2}\%$, (iii.) £1,265 15s. 10d for 77 days at $1\frac{7}{8}\%$, (iv.) £971 13s 6d for 81 days at 4%, (v.) £1,001 for 120 days at $4\frac{1}{2}\%$, (vi.) £365 for 365 days at 12%, (vii.) £1,904 7s 9d for 56 days at $3\frac{1}{2}\%$, (viii.) £12 450 for 131 days at $2\frac{7}{8}\%$, (ix.) £612 3s 11d. for 47 days at $3\frac{3}{4}\%$, (x.) £9,387 for 200 days at 5%

4. A sum of £1,050 is left on deposit for 9 mos. at $4\frac{1}{2}\%$. What is its amount at the end ?

5. Find simple interest on £358 12s from May 10th to Oct. 14th at 3%.

6. Find simple interest on £147 16s. 6d. from June 18th to Nov. 10th at $3\frac{1}{2}\%$.

7. What is the simple interest on £505 2s. 6d. for 5 years at $4\frac{1}{2}\%$ per annum ?

8. A man borrowed £280 and repaid it 9 months later ; find how much he repaid, interest being charged at $4\frac{3}{4}\%$.

9. Find the charge for a loan of £266 5s. for 4 mos. at $5\frac{1}{3}\%$.

10. Find the simple interest on £164 18s. from Nov. 12th, 1904, to April 17th, 1905, at 5%

11. Find the simple interest on £3,591 6s. 10d. at $3\frac{1}{3}\%$ from Feb. 1st to May 13th.

12. Find the simple interest on £685 from April 11th to Dec. 7th of the same year at $3\frac{1}{2}\%$ per annum.

13. The balance of my bank account at Dec. 31st, 1904, was £250 6s. 3d. The amounts paid in during the first half of 1905 were—Jan. 25th. £10 12s. 6d.; Feb. 7th, £25 6s.; April 28th, £73 6s. 8d.; the withdrawals for same period were—Jan. 4th, £5 ; Jan. 27th, £7 10s.; March 3rd, £15 15s. 10d.; May 10th, £17 ; June 14th, £75 5s. Interest is allowed on daily balances at $2\frac{1}{2}\%$ per annum. Find balance at June 30th.

14. A debtor owed me £71 10s. 6d. He paid it when it was 10 months overdue. If I charged him 5% interest, how much did I receive ?

CHAPTER XI

COMPOUND INTEREST: CALCULATION

If a sum of money, say £500, is borrowed for one year at 3%, the interest due at the end of the year is £15. To repay the principal and interest will require £515. This amount we saw in a previous chapter is called the Amount at end of one year. If now the £500 were borrowed for another year, at the end of the second another £15 would be due for interest; so that the interest for two years is £30. This would be at Simple Interest; but there are many cases in which the interest is not paid at the end of the first year, but the borrower retains (i.e., borrows for another year) the £515. In this case he will pay interest on the £515, so that the second year's interest will be £15 9s., and the amount due at end of two years so reckoned would be £30 9s. And in general we reckon interest for periods longer than a year according to this second method, known as the method of Compound Interest.

Thus, if at the end of each year (or period) the interest is added to the principal, the whole sum counting as principal for the next year (or period) and so on, the capital is spoken of as accumulating at Compound Interest. The Interest must then be calculated for each period, added to the principal at the beginning of that period, then the interest calculated on this new principal, and so on. It will be seen that this involves the repetition of the process of simple interest. The use of decimals and application of place value of digits will reduce the labour considerably. The following method will be found convenient for periods not too numerous. In a later chapter methods will be shown for this calculation for long periods.

Example : Calculate what £5,250 will amount to in 4 years at 3% per annum.

£
5250·
157·50
5407·50
162 2250
5569·7250
167·0918
5736·8168
172 1045
£5908·9113 = £5908 18s. 3d to nearest 1d.

£5,250 is the principal at commencement; 3% = ·03; therefore we multiply by 3, placing the figures of the product two places to the right. This gives £157·50 as first year's interest. This we add to the principal, and obtain £5,407·50 as the amount at the end of the first year. The student will notice how the proper placing of the product by ·03 allows the addition to be easily performed. A similar multiplication of £5407·50, as that is the principal for the second year, gives £162·2250 as the second year's interest, and with this added we have £5569·7250 as the amount at end of second year, or principal for third year. In the next multiplication by ·03 we begin with the digit 2, as no figures will be required beyond the fourth decimal place. As $5 \times 3 = 18$, we carry 2. Then $2 \times 3 + 2 = 8$, which is our fourth figure.

Similarly in the fourth multiplication, we say $6 \times 3 = 18$, carry 2, $1 \times 3 + 2 = 5$, put 5 in the fourth place.

If the interest for the four years is required, we subtract the original principal from the final amount, as the difference will be the increase, or the interest.

£5908·9113
less 5250

£658·9113 = £658 18s. 3d.

the interest for four years.

The student should now work the following example :—Find amount and interest of £327 18s. 4d. for 3 years at 4% per annum.

£327·9166	×	·04
13·1166		
341·0332	×	·04
13·6413		
354·6745	×	·04
14·1870		
368·8615	=	£368 17s 3d. Amount
327·9166		
40·9449	=	£40 18s. 11d Interest.

The final results need be only correct to nearest 1d.

Also :—Compound interest on £455 10s. 6d. for 3 years at $2\frac{3}{4}\%$ per annum.

Take $2\frac{3}{4}\% = \cdot 0275$.

£455·525		
9·1105	=	·02 times
3·1886	=	·007 times
·2278	=	·0005 times
468·0519		
9·3610		
3·2764		
·2340		
480·9233		
9·6185		
3·3664		
·2405		
494·1487	=	£494 3s.
455·525		
48·6237	=	£48 12s. 6d.

In the first multiplication by ·0275, note that we begin with 2 and use it on the 2 or second decimal figure in principal : $5 \times 2 = 10$, carry 1, $2 \times 2 + 1 = 5$, place 5 in fourth place. Then we use the 7 as multiplier on the 5 or first decimal place, as three places due to ·007 will move it to fourth place ; $2 \times 7 = 14$, carry 1, $5 \times 7 + 1 = 36$, put 6 in fourth place and carry 3, and so on.

The multiplication might also be done as follows :—

$$\begin{array}{l}
£455\ 525 \times \cdot 02\tfrac{3}{4}\ (= \cdot 02\tfrac{1}{2} + \tfrac{1}{4}). \\
\quad 9\ 1105 \\
\quad 2\cdot 2776 \\
\quad 1\cdot 1388 \\
\hline
468\ 0519, \ \&c
\end{array}$$

In taking multiplier $\cdot 00\tfrac{1}{2}$ we divide £455 525 by 2, moving result two places to the right. Then $\tfrac{1}{4}$ is $\tfrac{1}{2}$ of $\tfrac{1}{2}$—hence the last line of multiplication. The first method is the more generally applicable, but the fractional method often simplifies the labour.

Example : Required compound interest on £1,475 at $3\tfrac{1}{2}\%$ per annum for 2 years.

$$\begin{array}{ll}
£1475 & \times \cdot 03\tfrac{1}{2} \\
\quad 44\ 25 & \\
\quad\ \ 7\cdot 375 & \\
\hline
1526\cdot 625 & \\
\quad 45\ 7988 & \\
\quad\ \ 7\cdot 6331 & \\
\hline
1580\cdot 0569 & = £1580\ \ 1s.\ 2d \\
1475 & \\
\hline
\ \ 105\ 0569 & = £105\ \ 1s\ \ 2d
\end{array}$$

Example : Required compound interest on £306 10s. for 2 years at $2\tfrac{1}{2}\%$ per annum.

$$\begin{array}{ll}
£306\cdot 5 & \times \tfrac{1}{40}\ (\text{as } 2\tfrac{1}{2}\% = \tfrac{1}{40}). \\
\quad 7\ 6625 & \\
\hline
314\ 1625 & \\
\quad 7\ 8540 & \\
\hline
322\cdot 0165 & = £322\ \ 0s.\ 4d \\
306\cdot 5 & \\
\hline
\ \ 15\ 5165 & = £15\ \ 10s\ \ 4d.
\end{array}$$

Here we divide by 4 and move one place to the right. Similarly $5\% = \tfrac{1}{20}$.

In banks the interest is generally added every half-year. In such a case we take half a year as the period, but of course only half the rate of interest.

If a man deposits £365 in a bank on January 1st, 1905, what will it amount to at June 30th, 1906, if the rate of interest is $4\tfrac{1}{2}\%$ per annum ?

The interest per half-year $= 2\frac{1}{4}\% = \cdot 02\frac{1}{4}$.

```
£365·
   7·30
    ·9125
 373·2125 amount at June 30th, 1905.
   7·4642
    ·9330
 381·6097 amount at December 30th, 1905.
   7·6322
    ·9540
 390·1959 amount at June 30th, 1906
=£390 3s 11d.
```

It will be noticed that the interest is greater than if it had been calculated at $4\frac{1}{2}\%$ per year. Suppose we lend £100 at a nominal rate of $4\frac{1}{2}\%$ per annum, but interest added half-yearly, what will be the amount at the end of a year?

```
 £
 100        × ·02¼ for a half year.
   2
    ·25
 102·25
   2·0450
    ·2556
 104·5506
```

Showing an increase for the year of $4\cdot5506\%$. This rate is called "the effective annual rate."

Similarly the rate of interest on British Consols is nominally $2\frac{1}{2}\%$ per annum, but the interest is paid quarterly. What is their effective rate?

```
£100·       × 1/160 (= 1/4 of 1/40).
     ·625
 100·625
     ·6289
 101·2539
     ·6328
 101·8867
     ·6368
 102·5235
```

The effective rate $=$ £2 10s. 6d.% nearly.

Example: Find compound interest on £205 at $2\frac{3}{4}\%$ for $2\frac{1}{2}$ years

£		
205·	×	·02¾
4·10	=	·02 times
1·025	=	·00½ times
·5125	=	·00¼ times.
210·6375	×	·02¾
4·2127		
1·0531		
·5265		
216·4298	×	·01⅜
2·1643	=	·01 times.
·5410	=	·00¼ times
·2705	=	·00⅛ times
219·4056		£219 8s 1d
205		
14·4056	=	£14 8s 1d

The interest for the last year is, of course, at the rate of $\frac{1}{2}$ of $2\frac{3}{4}\%$, i.e., $1\frac{3}{8}\%$.

There are other questions which involve a similar principle where subtraction can be used instead of addition. For example, in estimating the value of a trader's stock or a manufacturer's machinery, an allowance is generally made for depreciation. One method is to calculate the depreciation at so much per annum on the value of such at the beginning of each year.

Example: The original cost of some machinery was £1,500. It is arranged to depreciate it each year at $8\frac{1}{3}\%$ of its value at beginning of each year. What will be its value at the end of 3 years? $8\frac{1}{3}\% = \frac{1}{12}$.

£	
1500	
125	
1375	by subtraction.
114·5833	
1260·4166	
105·0347	
1155·3819	= £1155 7s. 4d.

When the division cannot conveniently be performed in one division, the working may be arranged thus suppose the rate of depreciation in last question had been $10\frac{1}{2}\%$ for 2 years.

```
 £
1500
 150
   7.5

1342.5
 134.25
   6.7125

£1201.5375  = £1201 10s. 9d.
```

Here we perform the addition of £150 and £7·5, and subtraction from £1,500 at once See Chap. I

Again, we may add each year not only the interest, but also another instalment of principal, as follows

Example. A man saves each year £55, which he deposits at the end of each year at compound interest, 3% per annum. What will be total amount at end of 5 years?

```
  £55        Amount at end of 1 year
    1·65     Interest for 1 year
   55·       Second instalment

  111·65     Amount at end of 2 years.
    3·3495
   55·

  169·9995   Amount at end of 3 years
    5·1000
   55·

  230·0995   Amount at end of 4 years.
    6·9030
   55·

  292·0025   Amount at end of 5 years
= £292.
```

This example contains the principle of a sinking fund or annuity—a subject to be treated more fully in a later chapter.

EXAMPLES XI

Find the amount and interest of the following sums, compound interest :—

	Principal £	s	d	Rate % per Annum	Number of Years
1	1,000	0	0	2%	3
2	187	11	8	1%	5
3	2,095	10	0	$1\frac{1}{2}$%	4
4	3,475	12	6	$2\frac{1}{2}$%	2
5	568	18	4	3%	3
6	12,500	0	0	4%	3
7	752	15	0	$2\frac{1}{4}$%	2
8	365	0	0	$2\frac{3}{4}$%	3
9	10,000	0	0	5%	5
10	3,782	17	6	$3\frac{1}{2}$%	3
11	845	0	0	$1\frac{7}{8}$%	2
12	10	0	0	5%	10
13	6,250	0	0	$4\frac{1}{2}$%	3
14	1,333	6	8	4%	$3\frac{1}{2}$
15	999	0	0	9%	$1\frac{1}{2}$
16.	20,000	0	0	$2\frac{1}{2}$%	$2\frac{1}{2}$
17	567	12	10	$3\frac{1}{4}$%	2
18.	600	0	0	$3\frac{3}{4}$%	$2\frac{1}{2}$

19 One person lends £1,600 for 3 years at $2\frac{1}{2}$%; another lends £1,600 for 3 years at 3%; find how much more interest one gains, reckoning compound interest.

20 Find the value of £11 413 × $(1{\cdot}025)^4$ to the nearest shilling.

21. Find the accumulated value at end of 4 years of an annual payment at beginning of each year of £300, reckoning compound interest at 4% per annum

22 A man saves £150 every year, which he invests at the end of the year at 5% compound interest Find the amount of his savings at the end of the fourth year

23. A person borrows £150 at 4%. At the end of each year he pays £40. How much is he indebted at the end of 4 years (compound interest)?

24. A person borrows £100, and pays at the rate of 5% per month. If the interest is added on each month, and interest charged on new principal, what will the debt amount to at the end of the year?

25. The population of a town increases yearly by 18 per 1,000. What will it be at the end of 5 years?

26. A company borrows £10,000. Each year they pay £1,500 to reduce the principal, and pay interest at $4\frac{1}{2}$%. How much remains unpaid at end of 5 years?

27. If nominal rate of interest is 5% per annum, convertible quarterly, what is the effective rate?

28. A sum of £50 is deposited on a boy's sixteenth birthday to accumulate at $2\frac{1}{2}$% per annum. What will be the amount to his credit on his twenty-first birthday?

29. The population of a rural district decreases each year by 4% of its number at the beginning of the year. Find what it will be at end of five years if it is now 397,565.

30. Depreciate plant at 10% per annum. What will be value of plant at end of 4 years which is now valued at £6,000?

31. Show that an annual payment of £45,121 17s. 10d. for 4 years will pay off a debt of £160,000 borrowed at 5% per annum.

CHAPTER XII

SIMPLE INTEREST: INVERSE QUESTIONS, INCLUDING DISCOUNTS

There are four quantities involved in questions of Simple Interest: Principal, Interest, Rate, Time. Principal with interest added thereto is called the Amount.

In Chapter X it has been shown how to calculate the simple interest when the Time, Rate, and Principal are given. It will now be shown how to calculate any of the others from given particulars.

1.—To find rate per cent. per annum:—The particulars which must be known are the Principal, the Interest, and the Time. The method is the same as was used for finding a rate per cent. in Chapter IX.: we express the rate of the interest to the principal as a fraction, reduce this, of course, to the rate for a year, and change to a percentage.

Example: At what rate per cent. per annum will £750 yield £60 of interest in $1\frac{1}{2}$ years?

$$\text{The rate} = \frac{60}{750} \times \frac{1}{1\frac{1}{2}}$$

$$= \frac{\overset{2}{\cancel{6}}}{\underset{3}{\cancel{75}}} \times \frac{2}{\cancel{3}} \times \overset{4}{\cancel{100}}\%$$

$$= 5\tfrac{1}{3}\%.$$

The numerator of the fraction is 60 representing the given interest: the denominator is 750, representing the principal. This gives the rate for the period, but as the rate per annum is required, the fraction has to be divided by the number of years, $1\frac{1}{2}$.

Example At what rate per cent. per annum simple interest will £862 10s. amount to £1,037 3s. $1\frac{1}{2}$d in $4\frac{1}{2}$ years.

The interest is not given directly but it is included in the amount. By subtracting the principal from the amount, we get the interest—in this case £174 13s. $1\frac{1}{2}$d.

$$\therefore \text{ rate } = \frac{\overset{19.40625}{\cancel{174.65625}}}{862.5} \times \frac{2}{9} \times 100\% \text{ per annum.}$$

$$= \frac{19.40625 \times 2}{8.625}\% \text{ per annum} \qquad 8.625\overline{)38.81250}(4.5 \quad 43125$$

$$= 4\tfrac{1}{2}\%$$

13/$1\frac{1}{2}$

$$\text{Or rate} = \frac{174\frac{21}{32}}{862\frac{1}{2}} \times \frac{2}{9} \times 100\% \text{ per annum}$$

$13\frac{1}{8}$s.

£$\frac{105}{160}$

£$\frac{21}{32}$

$$= \frac{\overset{\overset{\overset{9}{\cancel{207}}}{\cancel{621}}}{\cancel{5589}}}{\underset{2}{\cancel{32}}} \times \frac{\cancel{2}}{\underset{\underset{\cancel{25}}{\cancel{575}}}{\cancel{1725}}} \times \frac{\cancel{2}}{9} \times \overset{4}{\cancel{100}}\% \text{ per annum}$$

$= 4\frac{1}{2}\%$ per annum.

Example: On 28th March, a sum of £225 was deposited. The depositor on 21st August withdrew the principal and interest, receiving £228 3s. What was the rate per cent. per annum? The principal is £225. Interest is £3 3s. The time is 146 days or $\frac{2}{5}$ of a year.

$$\therefore \text{ rate } = \frac{3.15}{\underset{9}{\cancel{225}}} \times \frac{5}{\cancel{2}} \times \overset{\overset{2}{\cancel{4}}}{\cancel{100}}\% \text{ per annum.}$$

$$= \frac{31.5}{9}\% \text{ per annum.}$$

$= 3\frac{1}{2}\%$ per annum.

2. To find the Time: —The particulars required are Principal, Rate, and Interest.

We know the interest for the period; we calculate the interest for one year; a simple division must give the time in years.

Example: In what time will £750 yield £45 of interest at 3 per cent. per annum.

$$\text{Total Interest} = £45$$

$$\text{Yearly Interest} = £750 \times \frac{3}{100}$$

$$= £22.5.$$

$$\text{No. of years} = \frac{45}{22.5} = 2$$

3. To find the Principal. —The particulars required are (*a*) Interest, Rate, Time, or (*b*) Amount, Rate, Time.

The method of reversing the fraction is the one advised. In case (*a*) we calculate the fraction to produce interest from a given principal; then multiply given interest by the fraction inverted.

Example: Find the principal to yield £63 7s. in four years at 5 per cent. per annum simple interest.

$$\text{The interest fraction} = \frac{5}{100} \times 4 = \frac{1}{5}$$

$$\text{Principal} = £63\ 7s. \times 5 = £316\ 15s.$$

In case (*b*) we calculate the interest fraction as in (*a*); from this we form the amount fraction, which we use inverted, operating on the amount.

Example: What sum will amount to £769 10s. in 2 yrs. and 4 mos. at 6% simple interest?

$$\text{Interest fraction} = \frac{6}{100} \times 2\tfrac{1}{3} = \frac{\overset{2}{6}}{100} \times \frac{7}{3} = \frac{14}{100}$$

$$\therefore \text{Amount fraction} = \frac{114}{100} = \frac{57}{50} \quad \left[\text{i.e. } 1 + \frac{14}{100} \right]$$

$$\therefore \text{Principal} = £769.5 \times \frac{100}{114} = £\frac{76950}{114} = £675.$$

DISCOUNT

From the previous examples we may conclude that a sum of money can have varying values, according to time. The general idea of interest is connected with depositing money in a bank, or of a formal loan: but in business all money, the immediate use of which we sacrifice, is expected to return with interest at some future date. We may invest it in a company, or use it in our business, but we expect a return of interest for it. Thus a present sum of money has in the future an increased value. Similarly, a sum to be received at some future date has a present value, which is, of course, less than the future value, owing to the interest expected. This arises in business chiefly in connection with what are known as Bills of Exchange. To put it briefly, these are written promises to pay a definite sum of money at some definite or determinable future date. An example of how they may arise is as follows:

A wholesale merchant—call him A. Merchant—is asked to supply certain goods of the value of £200 to a retail merchant—T. Smith. The latter instead of paying for the goods at the time agrees to pay the £200 in 3 months from the date. This promise is made in writing after this form:—

Bradford, 10*th March*, 1905.

£200.

Three months after day pay to my order Two Hundred Pounds.

To Mr. T. Smith.

A. Merchant.

The paper is stamped at rates varying according to the amount. At this stage A. Merchant is said "to draw on T. Smith." The latter now shows his agreement by writing across the face of the bill the words "Accepted—T. Smith." The student will now see the definite promise of Mr. T. Smith to pay £200 on a definite date, viz., 3 months from 10th March, 1905. There is a custom of allowing "3 days of grace," so that the payment is due on June 13th, 1905.

But it may happen that Mr. A. Merchant wishes cash before that date. To that end these documents are made negotiable, and he can sell this bill. A banker, or broker, if assured of the financial position of the parties—A. Merchant

and T. Smith—will generally buy such bills. Now it is clear that the banker or broker will not give £200 for a bill which gives them the right to receive £200 later. The practice is to calculate interest on £200 from the time of purchase till the time of payment, to deduct this interest from the amount, and pay the net amount, which is called its cash value. The points of arithmetical importance as regards these Bills of Exchange are :—

(1) The amount of the bill.

(2) The date of drawing.

(3) The due date calculated from (2) by adding on the term of the bill and the 3 days of grace.

(4) Rate of interest charged—called the rate of *discount.*

(5) The date on which it is sold—the technical name for such being "discounting."

Example : A bill for £730 drawn 17th March at 6 months is discounted August 3rd at 4%.

Date when drawn, 17th March.

∴ Due date, 20th September (including three days of grace).

Date of discounting, 3rd August

∴ No. of days = 48 days.

i.e., 48 days from 3rd August to 20th September.

$$\therefore \text{ The discount } = £730 \times \frac{48}{365} \times \frac{4}{100}$$

= £3·840

= £3 16s. 10d.

to nearest 1d

WORKING.

```
00730 ×      × 384
2 1900
  5840
   292
2 8032
  9344
   934
    93
3 840
```

The working is by method of "Third, tenth, tenth"

See Chapter on Simple Interest.

∴ The Cash Value of Bill = £726 3s. 2d.

Note : Discounts are calculated to the nearest 1d. Sometimes bills are drawn at 30 days "after sight," &c. The date of payment is then calculated, not from the date of drawing, but from the date of accepting.

Example: A bill of £254 17s. 6d. accepted September 10th at 60 days' sight, is discounted October 4th at 3%. What is its cash value?

60 days from September 10th + 3 days of grace = November 12th.

No. of days from October 4th to November 12th = 39.

$$\therefore \text{Discount} = £254{\cdot}875 \times \frac{39}{365} \times \frac{3}{100}$$

$$= £{\cdot}817$$

$$= 16s.\ 4d.$$

Cash Value = £254 1s. 2d.

WORKING

·0025,4875 × 234

5097
764
102
5963
·1987
199
20
817

The discount on a bill is thus seen to be simply the interest on the amount named in the bill. This is often called in books "Commercial" or "Bankers' Discount." There is a theoretical discount, known as "True Discount" or "Mathematical Discount," but we do not propose to discuss it in the present chapter. The discount, as illustrated in this chapter, is the only discount met with in commercial transactions.

The rate of discount is evidently just a rate of interest. It varies from time to time. The Bank of England publish their rate of discount—known as Bank Rate—every week. The market rate is generally lower than Bank Rate, but rises and falls with it.

EXAMPLES XII

1. Find rate of interest charged in the following cases:—

	Principal	Interest.	Time.
i.	£250	£31 5s.	4 years
ii.	£540	£12 3s.	$4\frac{1}{2}$ months
iii.	£15 10s.	19s. $1\frac{1}{2}$d.	6 months
iv.	£335 12s. 6d.	£3 7s. $1\frac{1}{4}$d.	3 months
v.	£532 5s. 10d.	£1 7s. $1\frac{1}{4}$d.	31 days

2. Find rate of interest in following cases

	Principal.	Amount.	Time
i.	£12	£13	$2\frac{1}{2}$ years
ii.	£925 16s 8d.	£1,152 13s 3d	$3\frac{1}{2}$ years
iii.	£235	£267 18s.	4 years
iv	£1,660	£1,684 4s 2d	7 months
v	£19,314 11s 8d	£19,354 16s	19 days

3 Find the time in the following cases –

	Principal	Interest	Rate % per ann.
i	£216	£27	$2\frac{1}{2}$
ii.	£640	£4	3
iii	£765	£107 2s	4
iv	£608 6s 8d	£1 1s	3
v	£41 13s 4d	£1 5s	$4\frac{1}{2}$

4 What sum will produce £6 13s of interest in 95 days at $2\frac{1}{2}\%$ per annum ?

5. At 3% for 4 years the interest is £12, find the principal.

6 What sum will yield £20 16s 6d. in 3 years at $3\frac{1}{2}\%$ per annum ?

7 What sum must I deposit in a bank at 4% to receive back £100 at end of 9 months ?

8. What sum will amount to £806 5s. 5d. in 3 years at $2\frac{1}{2}\%$ per annum ?

9 If £225 2s 4d. is the amount after 6 months at 3% per annum, simple interest, what was the principal ?

10. A sum of money was deposited on 28th March at $3\frac{1}{2}\%$ On 21st August the amount to be withdrawn was £228 3s. What was the sum deposited ?

11. What sum will amount to £4,724 14s 2d in $1\frac{1}{4}$ years at $3\frac{1}{2}\%$ per annum, simple interest ?

12 At what rate per cent. per annum, simple interest, must a sum be lent in order that the interest on it for $5\frac{1}{4}$ years may be ·168 of the sum lent ?

13. A sum of £78 was deposited in a bank on the 18th of March, and on the 11th of March it had gained £2 12s of interest. What was the rate per cent. per annum ?

14. If the interest on £650 for 5 months is £12 3s. 9d , what is the rate per cent ?

15. I invest £558 in a company and receive at the end of a year £37 15s. ; what rate of interest is that ?

16. Find the sum of money on which the simple interest for 9 months at 6% per annum is £3 7s. 6d

17 A money lender charges £6 5s 5d. for the loan of £86 for 7 months; what rate per cent. per annum is this?

18 A bill for £676 is drawn on March 8th for 3 months, it is discounted on April 15th at $4\frac{1}{2}\%$ per annum What does the holder receive for it?

19 A sum of money was deposited at $3\frac{1}{2}\%$ on 28th August, 1904, when the principal and interest were withdrawn on the 21st January, 1905, the depositor received £456 6s What was the sum deposited?

20. What principal if put out at simple interest for $3\frac{1}{2}$ years at $2\frac{3}{4}\%$ will amount to £554 13s?

21. Find the cash values of the following bills:—

i. Dated April 3rd for £604 10s 8d at 3 months and discounted April 12th at $2\frac{3}{4}\%$

ii. Dated August 7th for £755 at 2 months and discounted September 1st at $1\frac{7}{8}\%$

iii. Dated May 12th for £1,014 17s 6d at 3 months and discounted May 25th at $3\frac{3}{4}\%$

iv. Accepted March 22nd for £968 10s at 60 days' sight and discounted April 3rd at $2\frac{5}{8}\%$.

v. Accepted November 30th for £330 16s 8d. at 90 days' sight and discounted December 20th at 3%

22. A banker discounts a bill for £1,000, which has 2 months to run, at 3% discount What rate of interest is he receiving on the money he advanced? [Answer to two places of decimals]

23 A bill for £649 is dated April 3rd at 6 months, and is discounted on May 8th at $3\frac{1}{4}\%$ What does the banker charge?

24. The cash price of goods being £450, for what sum should we draw a 3 months' bill on the purchaser, assuming that we can immediately discount the bill at $3\frac{1}{2}\%$ discount?

25. A bill of £750, dated January 1st, 1905, at 6 months, is discounted on February 8th at 4%, what does the banker retain? If the above bill is dishonoured on presentation, for what amount should another be drawn at 3 months with interest charged at $4\frac{1}{2}\%$?

26 Find what the holder receives for a 3 months' bill for £850, dated 25th March, 1905, which a banker discounts on 10th April at $2\frac{7}{16}$ per cent per annum.

27. A loan of £500 is granted for 3 months at 5% per annum, and at the end of 3, 6, and 9 months it is renewed, an additional charge of $\frac{1}{4}\%$ being made on each renewal. Find the total sum paid by the borrower during the year, and calculate the actual rate per cent. per annum his loan has cost him.

28. A banker discounts a 3 months' bill for £1,250 at $2\frac{7}{8}\%$ on the day on which it is drawn. What rate of interest does he receive on the money he advances?

CHAPTER XIII

PROPORTION DIVISION MIXTURES

To divide a quantity into a number of equal parts, say 3, involves only a division by the number, i.e., 3.

Divide £189 among three persons equally. Each will receive $\frac{1}{3}$ of £189 = £63. But if the parts are to be unequal some condition must be given. A very common case of unequal division occurs when it is required to divide a quantity into parts proportional to given numbers.

Example: Divide £189 among three persons so that the shares will be proportional to the numbers 2, 3, 4. We consider this as a division into nine equal parts ($2+3+4=9$) and then a regrouping of the parts into one group of two of the equal parts, one group of three of the equal parts, and one group of 4 of the equal parts. Thus the fractions corresponding to the shares are $\frac{2}{9}$, $\frac{3}{9}$, $\frac{4}{9}$, and therefore the shares are £42, £63, £84.

The method, then, is to form a set of fractions, each having a denominator equal to the sum of the given numbers, and numerators respectively equal to the given numbers

Example: If 15s. is to be shared among three boys in proportion to their ages, which are 7, 6, and 5 years, how much should each get?

$$\text{The fractions will be } = \frac{7}{18}, \frac{6}{18}, \frac{5}{18}.$$

$\therefore$ The shares will be = 5s. 10d., 5s., 4s. 2d.

In practice the numbers composing the ratio should be brought to their simplest form before forming the fractions. The ratio will be the same if *all* the numbers are either divided by the same number or multiplied by the same number.

Example: Divide £900 in proportion to 2,400, 3,600, 4,800. Here the ratio can be divided by 1,200, giving 2.3.4.

The fractions are $\frac{2}{9}, \frac{3}{9}, \frac{4}{9}$

And the shares are £200, £300, £400

Example: Divide 951 in the proportion of $1\frac{1}{2}$, $1\frac{1}{3}$, $1\frac{1}{4}$, $1\frac{1}{5}$. Multiply all the numbers of the proportion by 60 and we get 90, 80, 75, 72.

The fractions are $\frac{90}{317}, \frac{80}{317}, \frac{75}{317}, \frac{72}{317}$

. The numbers required are $951 \times \frac{90}{317}$, $951 \times \frac{80}{317}$, $951 \times \frac{75}{317}$

and $951 \times \frac{72}{317}$

or 270, 240, 225, 216.

It will be noted that we chose 60 as the multiplier, because it is the L.C.M. of 2, 3, 4, 5, and thus produces whole numbers. Care must be taken to get the proportion right.

Example: Divide £55 amongst A, B, C, D, so that A's share is to B's as 2 3, B's to C's as 4 5, and C's to D's as 3·4.

A's	B's	C's	D's
2 ·	3		
	4	5	
	·	3 ·	4

Here we have the ratios of each pair, but we must get the ratio of all four on same basis. If we alter B's number to 12, and C's to 15, we get the four:—

A	: B	C	: D.
8	12	: 15	: 20

In altering B's from 3 to 12, we multiplied by 4, and consequently had to take four times A's, i.e., 8; in altering

B's from 4 to 12 we multiplied by 3, thus getting 15 for C; in altering C's 3 to 15 we multiplied by 5, thus getting 20.

$$\therefore \text{ The fractions are } \frac{8}{55}, \frac{12}{55}, \frac{15}{55}, \frac{20}{55}$$

$\therefore$ The shares are £8, £12, £15, £20.

Example: A guinea is to be divided between two persons so that one may receive half-a-crown for every shilling which the other receives. How much does each receive?

The proportion = 5:2 (i.e., proportional to the number of sixpences in half-a-crown and a shilling).

$$\therefore \text{ The fractions are } \frac{5}{7}, \frac{2}{7}$$

$\therefore$ The shares are 15s., 6s.

Example: A certain quantity of tea is put into five boxes containing quantities proportional to 10 : 12 : 14 : 20 : 21. The last box contains 102 lbs. What was the total quantity of tea?

$$\text{The last fraction} = \frac{21}{77} = \frac{3}{11}$$

$\therefore$ (By principle of reversing fraction)

$$\text{total quantity} = \overset{34}{\cancel{102}} \text{ lbs.} \times \frac{11}{\cancel{3}}$$

$$= 374 \text{ lbs.}$$

$$= 3 \text{ cwts., } 1 \text{ qr., } 10 \text{ lbs.}$$

Proportional Division is often required in the matter of division of profits among the members of a partnership.

A partnership is the association of two or more persons, within a certain number fixed by law, who join together their money, labour, and skill for the purpose of carrying on a commercial undertaking in a community of profits and losses. Very often the profits or losses are to be divided amongst the

partners in proportion to the capitals advanced by each. The actual method of distribution is, however, a matter of arrangement.

Example: Three partners respectively invest in the business £4,730, £3,680, £2,840. What is each one's share of a profit of £595 6s. 3d ? As no special arrangement is mentioned, we take it that the division is to be proportional to the capitals, i.e., 4,730 : 3,680 : 2,840, or 473 : 368 : 284.

$$\therefore \text{ The fractions are } \frac{473}{1125}, \frac{368}{1125}, \frac{284}{1125}.$$

$$\therefore \text{ The shares} = £595 \text{ 6s. 3d.} \times \frac{473}{1125},$$

$$£595 \text{ 6s. 3d.} \times \frac{368}{1125}, \quad £595 \text{ 6s. 3d.} \times \frac{284}{1125}$$

$$\text{Now } £595 \text{ 6s. 3d.} \times \frac{473}{1125}$$

$$= £595\tfrac{5}{16} \times \frac{473}{1125}$$

$$= £\frac{\overset{127}{\cancel{9525}}}{16} \times \frac{473}{\underset{15}{\cancel{1125}}}$$

$$\begin{array}{r} 473 \\ 946 \\ 3311 \\ \hline 60071 \end{array}$$

$$= £\frac{60071}{240}$$

$$= £250 \text{ 5s. 11d}$$

$$\text{Second share} = £\frac{127}{\cancel{16}} \times \frac{\overset{23}{\cancel{368}}}{15}$$

$$\begin{array}{r} 254 \\ 381 \\ 15)\overline{2921} \\ \hline 194\text{—}11 \end{array}$$

$$= £194 \text{ 14s. 8d.}$$

$$\text{Third share} = £\frac{127}{\underset{4}{\cancel{16}}} \times \frac{\overset{71}{\cancel{284}}}{15}$$

$$\begin{array}{r} 889 \\ 127 \\ 60)\overline{9017} \\ \hline 150 \end{array}$$

$$= £150 \text{ 5s. 8d.}$$

Example: Three partners contribute respectively £438, £292, £730. They agree that each is to receive 5% on their capitals, and that any remaining profit is to be equally divided. Find each share when the gain is £200.

5% of £438 = £21 18s.

5% of £292 = £14 12s.

5% of £730 = £36 10s.

Total interest = £73.

Remaining profit = £127

∴ Each equal share = £42 6s. 8d.

∴ 1st partner's share = £64 4s. 8d.

2nd partner's share = £56 18s. 8d.

3rd partner's share = £78 16s. 8d.

This is quite a common arrangement, as it allows for differences in capital, and at same time gives an equal share of profit due to (presumably) equal share of work.

Sometimes in forming the ratio the element of time also enters.

Example: A, B, C engage in a business. A puts in £5,000, and after four months another £5,000; B puts in £3,000, and after six months adds £6,000; C puts in £3,500, but withdraws at end of eight months. How should the first year's profit—£10,000—be divided among them?

$$\text{A's number} = 5{,}000 \times 4 + 10{,}000 \times 8 = 100{,}000.$$

$$\text{B's number} = 3{,}000 \times 6 + 9{,}000 \times 6 = 72{,}000$$

$$\text{C's number} = 3{,}500 \times 8 = 28{,}000.$$

$$\therefore \text{ Ratio} = 100 : 72 : 28.$$

$$= 25 : 18 : 7.$$

$$\therefore \text{ The fractions} = \frac{25}{50}, \frac{18}{50}, \frac{7}{50}$$

∴ A's share = £5,000.

B's share = £3,600

C's share = £1,400.

Mixtures

There are two cases of mixtures which require consideration as regards the arithmetic . (1) when we mix given quantities of goods of different values, to find the value of the mixture.

Example : A wine merchant mixes two casks of wine at 12s. a gallon with one cask at 18s. a gallon. What is the value of the mixture per gallon ?

	2 gallons @ 12s.	= £1 4s.
	1 gallon @ 18s	= 18s.
∴	3 gallons of mixture	£2 2s
∴	1 gallon of mixture	= 14s.

Case 2.—When we have goods of different values, and it is required to find in what proportion they should be mixed to produce a mixture of a certain value.

Example : (Two ingredients.) We have two qualities of tea, worth 2s. 4d. and 2s. 11d. per lb. respectively. In what proportion should they be blended to give a mixture worth 2s. 8d. a lb. ?

Ingredients	28d.		35d.
Mean value		32d.	
This gives	4d. below,	3d above mean value.	

∴ The proportion = 3 : 4,

i.e., for every lb. of the former used we get 4d. below mean value, and for every lb. of latter we get 3d. above ; we have to choose quantities so that the amount above will be exactly equal to the amount below. In the case of two ingredients the numbers are evidently the reverse of the numbers which express amounts above and below.

Example : Spirit at 18s. 6d. per gallon is blended with spirit at 22s. ; compare the quantities used if the blend is worth 19s. 6d. per gallon.

Ingredients	18s. 6d.		22s.
Mean		19s. 6d.	
∴ (In sixpences) 2 below		5 above.	
∴ Ratio is		5 : 2	

Example: (More than two ingredients.) This is more difficult, inasmuch as the solution is indeterminate, i.e.. more than one solution is possible. Mix ale at 2s. 8d., 2s., 1s. 10d. per gallon with water, so that mixture may be worth 1s. 6d. per gallon.

Ingredients	2s. 8d.	2s	1s. 10d.	0s.
Mean		1s. 6d.		
∴ (In pence)	14 above	6 above	4 above	18 below.

∴ One set of numbers $= 1 : 3 . 1 : 2$

This gives $1 \times 14 + 3 \times 6 + 1 \times 4 = 36$ above.

And $2 \times 18 = 36$ below.

			s	d
Verification:	1 gallon @ 2s. 8d.	=	2	8
	3 gallons @ 2s.	=	6	0
	1 gallon @ 1s. 10d.	=	1	10
	2 gallons @ 0s	=	0	0
	∴ 7 gallons	=	10	6
	∴ 1 gallon	=	1	6

The ratio $9 : 3 : 9 : 10$ also satisfies, for $9 \times 14 + 3 \times 6 + 9 \times 4 = 180$ above, and $10 \times 18 = 180$ below.

The first set was obtained by linking 14 and 4 against 18, then 3 sixes against another 18.

The second set was obtained by linking 9 fourteens against 7 eighteens, 3 sixes against 1 eighteen, 9 fours against 2 eighteens, thus giving 10 eighteens.

But no rules can be given for such linkings; only consideration and experience can give facility in seeing what numbers should be chosen.

Sometimes other conditions are introduced into the questions.

Example: In what proportion must wines of 15s., 20s., 26s., and 36s. a gallon respectively be mixed so that the mixture may be sold at 24s. a gallon with $14\frac{2}{7}\%$ profit? It

must be noted that in the mixing we require the value of the mixture, not its selling price; therefore we must first find the cost price.

$$\text{The selling price fraction} = \frac{114\frac{2}{7}}{100} = \frac{8}{7}$$

$$\therefore \text{Cost price of mixture} = 21s \times \frac{7}{8} \text{ (reversing fraction).}$$
$$= 21s$$

Ingredients	15s	20s	26s	36s
Mean		21		
or	6 below	1 below	5 above	15 above.
∴ One solution	3	2	1	1
or another solution	5	5	1	2

Verification of first solution :—

		£	s	d
3 @ 15s	=	2	5	0
2 @ 20s	=	2	0	0
1 @ 26s	=	1	6	0
1 @ 36s	=	1	16	0
7	=	7	7	0
1	=	1	1	0

$$\therefore \text{Selling price} = 21s \times \frac{8}{7} = 24s$$

$$\text{or rate of profit} = \frac{3}{21} = \frac{1}{7} = 14\frac{2}{7}\%.$$

Example. A dairyman buys milk at $2\frac{1}{2}$d. per quart, dilutes it with water, and sells the mixture at 3d. per quart. His profits are 60% on outlay. How much water is there in each quart sold?

$$\text{Selling price fraction} = \frac{160}{100} = \frac{8}{5}$$

$$\therefore \text{Value of mixture per quart} = 3d \times \frac{5}{8} = 1\frac{7}{8}d$$

Ingredients	$2\frac{1}{2}$d	0d
Mean		$1\frac{7}{8}$d
∴ (In $\frac{1}{8}$ of 1d.)	5 above	15 below
∴ Ratio	3 :	1

∴ In every quart of milk sold there is $\frac{1}{2}$ a pint ($\frac{1}{4}$ of 1 quart) of water.

Example : A grocer having four sorts of tea at 1s. 9d., 2s., 2s. 6d., 2s. 9d. a lb., wishes a mixture of 81 lbs. at 2s. 4d. a lb. What quantity must he take of each sort ?

Ingredients	1s. 9d		2s.	2s. 6d.		2s. 9d.
Mean			2s 4d.			
	7 below		4 below	2 above		5 above
One solution	1		1	3	:	1
Another ,,	5		1 .	2	:	7
Another ,,	2	.	5 ·	7	.	4

If we take the first solution, we have now to divide 81 lbs. proportionately to 1 : 1 3 1.

$\therefore$ Quantity @ 1s. 9d. $= 81 \times \frac{1}{6}$ lbs $= 13\frac{1}{2}$ lbs

2s. $= 13\frac{1}{2}$ lbs.

2s. 6d $= 81 \times \frac{3}{6}$ lbs $= 40\frac{1}{2}$ lbs

2s 9d $= 13\frac{1}{2}$ lbs

If we take the second solution the

Quantity @ 1s 9d. $= 81 \times \frac{5}{15}$ lbs $=$ 27 lbs.

2s. $= 81 \times \frac{1}{15}$ lbs. $= 5\frac{2}{5}$ lbs.

2s. 6d. $= 81 \times \frac{2}{15}$ lbs. $= 10\frac{4}{5}$ lbs.

2s. 9d. $= 81 \times \frac{7}{15}$ lbs. $= 37\frac{4}{5}$ lbs

This would not be so convenient for measuring out as the first solution.

Example : A merchant has 4 lbs. of coffee at 1s. 10d., 6 lbs. at 1s. 4d. He wishes to use these up by mixing with them others at 1s. 8d., 1s. 3d., to make a mixture worth 1s. 5d. What quantities of the latter two must he take ?

Ingredients	1s. 10d.		1s. 4d.		1s. 8d.		1s. 3d.
Mean				1s. 5d			
$\therefore$	5 above		1 below		3 above		2 below
$\therefore$ Numbers are	4	:	6	:	2	:	10

Here only one solution is possible, as the first two numbers are fixed, giving $4 \times 5 - 6 \times 1 = 14$. Thus we have to make 14 out of 3 above and 2 below.

This method can be applied to questions on alcohol strengths.

Proof spirit is taken as a standard, and represented by 1. Alcoholic values are indicated by terms over proof (o.p.) and under proof (u.p.). The o.p. values are added as percentages to 1 and u.p. values subtracted; e.g., 20 o.p. is represented by 1·20; 5 u.p. by 1—·05, i.e., by ·95.

Example: Find strength of a mixture made from 7 gallons 10 o.p. with 8 gallons 16 u.p.

	7 @ 1·10	=	7·70
	8 @ ·84	=	6·76
i.e.	15	=	14·46
or	1	=	·96

i.e., 4 u.p.

Example: How many gallons of 6 u.p. must be mixed with 12 gallons 5 o.p. to bring mixture to 1 o.p.

Ingredients	·94	1·05
Mean	1·01	
∴	·07 below	·04 above
∴ Proportion	4	7

∴ To 12 galls. of 5 o.p. we must mix $12 \times \frac{4}{7}$ of 6 u.p

i.e., $6\frac{6}{7}$ gallons.

Another application is found in alloys. In goldsmiths' and silversmiths' work oz. troy is the standard weight, and only oz. troy and decimals of the oz. are used.

The old way of estimating the fineness of gold was by carats: 24 carats representing pure gold; 9 carat gold means a gold $\frac{9}{24}$ of its weight being pure gold. Fineness is also nowadays reckoned in millièmes (thousandths). Thus gold 915 fine means a gold $\frac{915}{1000}$ of its weight being pure gold.

Example: A goldsmith is required to make 20 ozs. of an alloy 12 carats fine. He has golds 10 and 15 carats fine. How will he mix these?

Ingredients	10		15
Mean		12	
∴	2 below		3 above
∴ Proportion	3	:	2

∴ Quantity of 10 carats $= 20 \text{ oz.} \times \frac{3}{5} = 12 \text{ oz.}$

Quantity of 15 carats $= 20 \text{ oz.} \times \frac{2}{5} = 8 \text{ oz.}$

The oz. troy contains 480 grains troy.

Example : Mix gold 886 fine and silver 520 fine to produce an alloy 840 fine.

Ingredients	886		520
Mean		840	
∴	46 above		320 below
∴ Proportion	160	:	23

Standard gold in England is 22 carats, i.e., $\frac{11}{12}$ fine, and its value is £3 17s. 10½d. an oz. at the mint.

Standard silver in England is 11 oz. 48 grains fine, i.e., in 12 oz. troy there are 11 oz. 48 grains of pure silver.

The fineness of a mixture is often expressed by stating how much better or how much worse it is than the standard.

Example : U.S. dollar worse 0·8½, weight 17 dwts. 8 grains. This means that instead of being 22 carats, it is 22—·8½, or 21·1½ fine.

∴ Weight of standard gold

$$= \frac{21{\cdot}15}{22} \text{ of } 17 \text{ dwt. } 8 \text{ grains.}$$

$$= \frac{\overset{1\cdot41}{\cancel{21{\cdot}15}}}{22} \text{ of } \frac{\overset{13}{\cancel{416}}}{\underset{15}{\cancel{480}}} \text{ oz.}$$

$$= \frac{18{\cdot}33}{22} \text{ of an oz.}$$

∴ Value $= £3\ 17\text{s.}\ 10\tfrac{1}{2}\text{d.} \times \frac{18{\cdot}33}{22}$

$= £3{\cdot}244$

$= \underline{£3\ 4\text{s.}\ 10\tfrac{3}{4}\text{d.}}$

```
    |3·89375| × 1·833
    |3·11500|
    |  ·11681|
    |   1168|
2·2 |7·13724|3·244
    |  53
    |   97
    |    92
    |     44
```

EXAMPLES XIII

1. Divide 780 proportionally to 1 . 2 : 3 : 4

2. Divide 28,561 into parts proportional to 1, 1, 4. 7.

3. Divide £720 into parts in ratio of 16 11.

4 Three parts of a mixture are as 3 4 . 5 ; what percentage is each of the whole ?

5. Divide £750 into parts in ratio 7 10 : 13

6. Divide £260 into parts proportional to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.

7. Divide 150 into parts proportional to 2, 3 2, 4 8.

8 Divide £4 2s 6d in ratio of 3 . 7

9 Divide £1,330 between three persons, so that their shares may be proportional to $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{5}$.

10 684 policemen are to be distributed among three towns, in proportion to their populations—viz , 10,944, 12,312, 25,992 How many will be sent to each town ?

11. Four villages have to raise £300 by contributions proportional to their rateable assessments, which are £3,750, £7,164, £2,946, and £2,140 respectively How much do they contribute severally, and what is the rate in the £ ?

12. Divide £46 4s. among three persons so that their shares are in the proportion of 7, 8, 13.

13 Divide £111 8s 11½d among A, B, C, so that B's share shall be $\frac{2}{3}$ of A's share, and C's share $\frac{1}{4}$ of B's share

14. A sum of money is divided between A and B in the following way : one-fifth is paid to A, and one-seventh of the remainder to B ; after this four-ninths of the remainder to A, and the rest to B. A receives £1 6s. 8d., what does B receive ?

15. Pure spirit can be bought at 14s. a gallon ; a wine merchant reduces this spirit with water until he gains 25% by selling his mixture at 12s. 6d per gallon ; in what ratio does he mix the spirit and water ?

16. How many lbs. of tobacco, at 5s. 3d. a lb., must a tobacconist mix with 4 lbs. at 6s. 6d. a lb., that he may sell the mixture at 7s. 10d. per lb. and gain $\frac{1}{3}$ of his outlay ?

17. A tea dealer buys a chest of tea containing 2 qrs. 17 lbs. at 3s. 1½d. per lb., and 2 chests, each containing 3 qrs. 7 lbs. at 3s. 5½d. per lb. What will he gain per cent. by selling the mixture at 4s. per lb. ?

18. A grocer buys two sorts of tea, at 1s. 4d. and 1s. 10½d. a lb. How must he mix them so that by selling the mixture at 2s. a lb. he may gain 20% ?

19. I have a pint of mixture consisting of 1 part of acid to 5 of water; how much water must I pour into it so that the acid may be $\frac{1}{20}$ of the water?

20. A spirit merchant mixes 40 gallons of whisky at 15s. 6d. per gallon with 48 gallons at 17s. 1d., and sells the mixture so as to gain 10%. At what price per gallon does he sell it?

• 21. A poor rate at 11d. in the £ is made up of the Union contribution 5½d., borough rate 4d., asylum, &c., 1½d. The rate on a certain property amounts to £100 8s. 5d.; how much of this is for the borough rate, and at what amount is the property assessed.

22. A man buys 30 lbs. of tea at 2s. 6d. a lb. and 20 lbs. at 3s. 6d.; he mixes them and sells the mixture at 3s. a lb. What is his gain per cent.? A grocer buys 15 lbs. of tea at 1s. 6d. per lb. and 21 lbs. at 2s. 6d. per lb.; he mixes them and sells at 2s. 2d. per lb.; find gain per cent.

23. A, B, C entered into partnership. A advanced £840 for 10 months, B £960 for 8 months, C £720 for a year. If the profits amounted to £883 4s. 6d., how much should each receive?

24. Two partners advance £1,000 each on starting a business. Three months afterwards they admit two additional partners, one advancing £1,200, the other £800. At the end of the year the profits, amounting to £2,100, are divided in proportion to their capital. Find the share of each partner.

25. Two partners advanced £500 and £650 respectively. The agreement is that each shall receive 4% on his capital, and the remaining profit be divided in proportion to capitals. Find each man's share of a profit of £600.

26. A possesses £1,200 of the capital of a firm; B the other partner possesses £2,000. A receives 10% of gross profits for managing—the rest being divided in proportion to capitals. Find each one's share of a gross profit of £800.

27. Standard gold is 22 carats fine and is worth £3 17s. 10½d. an oz. troy. Find the value in shillings and pence (to $\frac{1}{10}$ of a penny) of a 20 yen piece, $\frac{9}{10}$ fine, weighing 257·21 grains.

28. A goldsmith has to supply 90 grains of an alloy containing gold and silver in proportions 11 : 1. He has an alloy in proportion 15 : 1. How much of this alloy should he take, and how much silver should he add?

29. A person wishes to melt equal quantities of gold, 943 and 827 fine, with alloy so as to get a gold 467 fine. What quantities of each must he take?

30. William Foster is in partnership with his son John. The profits for the year came to £5,593 15s., which is appropriated as follows :

(a) £593 15s. is set aside to a reserve account.

(b) W. Foster is credited with a year's interest at 5% on his capital (£20,000).

(c) The balance is divided among the two partners in proportion of 7 : 3.

How much does each partner receive ?

31. A grocer sells his best quality of tea at 3s 6d. a lb. and thereby gains a profit of 5% ; he sells his second quality at 2s. 6d. per lb., gaining 8%. In what proportion does he mix the two qualities if, by selling the mixture at 2s. 9d. per lb., he gains 10% ?

32. Find proportions of these mixtures to sell at the given prices :—

(i) Spirits at 10s. 6d., 13s. 5d., 12s. 9d., 14s. 2d., 15s., to sell at 13s. a gallon.

(ii) Teas at 3s. 6d., 2s. 2d., 1s. 8d., 5s., 4s. 2d., to sell at 2s. 6d. a lb.

33. Two partners started in business, A contributing £12,000 and B £18,000. B was to have 15% of the profits for his salary as manager. At the end of 7 months A withdrew one-third of his capital, and 2 months later B withdrew one-half of his. The profits of the year amounted to £3,130. What sum of money ought each to receive ?

34. A, B, and C are partners in a business ; the profits are £825 16s. 6d. and they are to be divided thus :—A is to take £6 for every £4 10s. which B takes, and for every £3 3s. that C takes. What is the share of each ?

35. Find the alcoholic strength of a mixture of 13 gallons 5 o.p., 10 gallons 3 o.p., and 16 gallons 5 u.p.

36. Find price of 673·829 oz. gold, 910 fine, at £3 17s. 9d. per oz. standard.

37. One dollar weighs 412·5 grains and is $\frac{9}{10}$ fine : 1 lb. troy of British standard silver, $\frac{37}{40}$ fine, is coined into 66 shillings. How many pence are in 1 dollar if British standard silver is worth 5s. 1d per oz. troy ?

38. A gold chain of 18 carats weighs $2\frac{1}{2}$ oz., how much pure gold is there in it ? Also find its value if 1 oz. of standard gold (i.e., $\frac{11}{12}$ths fine) is worth £3 17s. 9d.

CHAPTER XIV

TABLES OF MEASURES

It is assumed that the student is familiar with the ordinary tables of English Weights and Measures. The principal ones are given here for reference. The Unit of Length is the *yard*, which is a length settled by Act of Parliament; its multiples and sub-divisions are as follows:—

Length.

12 inches	=	1 foot.
3 feet	=	1 yard.
22 yards	=	1 chain.
10 chains	=	1 furlong.
8 furlongs	=	1 mile.

It should also be remembered that 1 mile = 1,760 yards. In surveying the chain is sub-divided into 100 links.

Surfaces or areas are of two dimensions, and are expressed in terms of Square Measure.

Area.

144 sq. inches	=	1 sq. foot.
9 sq. ft.	=	1 sq. yard.
484 sq. yards	=	1 sq. chain.
10 sq. chains	=	1 acre.

Other useful equivalents are:—

4840 sq. yards	=	1 acre.
640 acres	=	1 sq. mile.

Acres are sometimes sub-divided into roods and sq. poles, thus:—

40 sq. poles	=	1 rood.
4 roods	=	1 acre.

Land is often thus measured.

Volume.

1728 cubic inches = 1 cubic foot.
27 cubic feet = 1 cubic yard.

For measuring liquids the gallon is the unit, and the table is as follows :—

Liquid Measure.

4 gills = 1 pint.
2 pints = 1 quart.
4 quarts = 1 gallon.

For the capacity in dry measuring the table contains in addition to above :—

2 gallons = 1 peck.
4 pecks = 1 bushel.
8 bushels = 1 quarter.

In weighing most substances the unit is the pound (lb.) avoirdupois.

Weight—Avoirdupois.

16 oz = 1 lb.
28 lbs. = 1 qr.
4 qrs. = 1 cwt. (hundredweight).
20 cwts. = 1 ton.

It is well also to know that

112 lbs. = 1 cwt., 14 lbs. = 1 stone.

Weight—Troy.

24 grains = 1 dwt. (pennyweight).
20 dwts. = 1 oz.
12 oz. = 1 lb. troy.

Troy weight is used in weighing gold and silver.

1 lb. avoirdupois contains 7,000 grains.
1 lb. troy = 5,760 grains.

Money.

4 farthings	=	1 penny (d.).
12d.	=	1 shilling.
20 shillings	=	1 pound (£).

The unit of value is the sovereign or pound. Out of 480 oz. troy of standard gold (i.e.. $\frac{11}{12}$ths fine) 1,869 sovereigns are coined.

If it is required to change quantities from one denomination to another, the figures given in the tables are available.

Example: Reduce 21 tons 5 cwt. 1 qr. 11 lbs. to lbs.

T.	cwt.	qr.	lbs.
21	5	1	11
20			
425			
4			
1701			
28			
3402			
13608			
11			
47639 lbs.			

To reduce tons to cwts. we multiply number of tons by 20, adding in the 5 cwts.; this result is now multiplied by 4 to bring cwts. to qrs.. adding in 1 qr.; this result is then multiplied by 28, and the 11 lbs. is added.

In ordinary practice the figuring may be shortened by omitting the multipliers:—

T.	cwt.	qr.	lbs.
21	5	1	11
425			
1701			
3402			
13608			
11			
47639 lbs.			

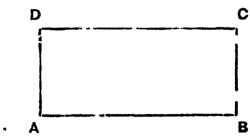
An expression containing cwts. qr. lbs. may also be worked thus :—

$$
\begin{array}{ccrcl}
\text{cwt.} & \text{qrs.} & \text{lbs.} & & \\
13 & 2 & 20 & & \\
 & & & & \\
 & & 1320 & = & 13 \times 100 + 20 \\
 & & 156 & = & 13 \times 12 \\
 & & 56 & = & 2 \times 28 \\
\hline
 & & \underline{1532} & &
\end{array}
$$

This could be done mentally.

Quantities in other measures can be similarly worked by use of the proper equivalents as given by the tables.

In measuring *Areas* the fundamental area is a rectangle, which is a figure of four sides with each of its angles right angles. To calculate the area of such a figure we measure two adjacent sides, as AB, AD (see figure), using the same



unit for each—say $AB = 1\frac{1}{8}$ inches, and $AD = \frac{1}{2}$ inch—the number of units of area is equal to the product of $1\frac{1}{8} \times \frac{1}{2}$ or $\frac{9}{16}$, and the unit of area will be sq. inch. So we say the area $= \frac{9}{16}$ sq. inch.

Care must be taken as to the denominations of the lengths. For ordinary practice it will be found useful to adopt a habit of working always in feet and square feet.

Example :—Find the area of a rectangle which measures 16 ft. 2 in. one way and 3 ft. 8 in. the other. The measurements of a rectangle are generally called its length and breadth.

$$\begin{aligned} \text{The area} &= 16\tfrac{1}{6} \times 3\tfrac{2}{3} \text{ sq. ft.} \\ &= \frac{97}{6} \times \frac{11}{3} \text{ sq. ft.} \\ &= \frac{1067}{18} \text{ sq. ft.} \\ &= 59\tfrac{5}{18} \text{ sq. ft.} \end{aligned}$$

$$\begin{array}{r|r|l} 18 & 1067 & 59 \\ & 167 & \\ & 5 & \end{array}$$

The student must be careful to remember that an area is always in square measure. When a rectangle measures the same each way it is called a square ; so that the unit of area —the square yard—is a rectangle measuring 1 yard each way.

The length and breadth may be called the factors of the area, and so if the area of a rectangle be known, and one of the factors be known, a division will give the unknown factor, it being remembered that the area is in square measure, the length and breadth each in ordinary length measure.

Example :—The area of a rectangle is 50 square yards. Its breadth is 15 feet. What is its length ?

$$\begin{aligned} \text{The length} &= \frac{\overset{10}{\cancel{50}} \times \overset{3}{\cancel{9}}}{\underset{\cancel{5}}{\cancel{15}}} \text{ ft.} \\ &= 30 \text{ ft} \\ &= 10 \text{ yds.} \end{aligned}$$

Great care must be observed as to the *names* at each step.

The area $= 50$ square yards $= 50 \times 9$ square feet.

Example : A room measures 18 ft. 10 in. by 15 ft. 8 in ; the carpet chosen for it is 27 inches wide, and costs 5s. 6d.

a yard. What length of carpet will be required, and what will be the total cost ?

$$\text{The area of the floor} = 18\tfrac{5}{6} \times 15\tfrac{2}{3} \text{ sq. ft.}$$

Now this evidently is also the area of the carpet to cover it, but we know the breadth of the carpet to be 27 in., or $\frac{27}{12}$ ft

$$\therefore \text{ length of carpet} = \frac{113}{6} \times \frac{47}{3} \times \frac{12}{27} \text{ ft.}$$

$$= \frac{113}{\cancel{6}} \times \frac{47}{3} \times \frac{\overset{2}{\cancel{12}}}{27} \times \frac{1}{3} \text{ yds.}$$

$$= \frac{10622}{243} \text{ yds.}$$

$$= 43\tfrac{173}{243} \text{ yds}$$

$$\therefore \text{ Cost} = \frac{\overset{5311}{\cancel{10622}}}{243} \times \frac{11}{\cancel{2}} \text{ sh.}$$

$$= \frac{58421}{243} \text{ sh.}$$

$$= \underline{\text{£}12 \text{ 0s. 5d.}} \text{ nearly.}$$

Questions on papering walls cause many pupils trouble owing to confusion over measurements. In these questions the measurements of the room are given—length, breadth, and height. The student should remember that the walls are surfaces, and have only two measurements ; therefore he must calculate the area of each wall separately, or imagine a single wall made of the four joined one after the other.

Example : The length, breadth, and height of a room are respectively 18 ft., 12 ft., 11 ft. Find the area to be papered. (It is assumed that the walls are all to be papered, i.e., there are no windows, doors, &c.)

First way : —The measurements of a wall the long way of the room are 18 ft. and 11 ft.

$$\therefore \text{ its area} = 198 \text{ sq. ft.}$$

$$\text{The area of an end wall} = 12 \times 11 \text{ sq. ft.}$$

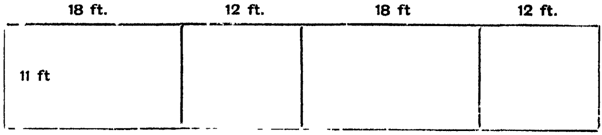
$$= 132 \text{ sq. ft.}$$

and there are two long walls and two end walls

$$\therefore \text{ Total area} = 198 + 198 + 132 + 132 \text{ sq. ft.}$$

$$= 660 \text{ sq. ft.}$$

Second way :—



Considering the four walls as forming one surface we find its length to be 60 ft. and its breadth 11 ft

$\because$ Area $= 60 \times 11$ sq. ft. $= 660$ sq. ft.

The second way is preferable in working.

Example : A room has 18 ft. 6 in. in length, 12 ft. 3 in. in breadth, and 10 ft. 9 in. in height. A "piece" of paper is 21 in. wide and 12 yds. long. How many pieces will be required, counting a part of a piece as a whole ; and what will be the charge at 1s. $4\frac{1}{2}$d. a yd. ?

The length of four walls considered as one is 2(18 ft. 6 in. + 12 ft. 3 in. = 61 ft. 6 in

$$\therefore \text{ Area of walls} = 61\frac{1}{2} \times 10\frac{3}{4} \text{ sq. ft.}$$

$$\text{Area of one piece} = \overset{3}{\cancel{36}} \times \frac{21}{\cancel{12}} \text{ sq. ft.} = 63 \text{ sq. ft.}$$

$$\therefore \text{ No. of pieces} = \frac{\overset{41}{\cancel{123}}}{2} \times \frac{43}{4} \times \frac{1}{\underset{21}{\cancel{63}}}$$

$$= \frac{1763}{8} \qquad\qquad \begin{array}{r} 43 \\ \underline{172} \end{array}$$

$$= 220\frac{3}{8}$$

$$= 221 \text{ pieces.}$$

$\therefore$ Cost $=$ 1s. $4\frac{1}{2}$d. $\times$ 221	@ 1s.	£11	1s. 0d.
	@ $\frac{1}{4}$s.	2	15s. 3d.
$=$ £15 3s. $10\frac{1}{2}$d.	@ $\frac{1}{8}$s.	1	7s. $7\frac{1}{2}$d.

When the walls contain windows, &c., their areas must be calculated and subtracted from the area of the four walls before the number of "pieces" is calculated.

Example: The length, breadth, and height of a room are respectively 24 ft. 5 in., 18 ft. 7 in., 15 ft. 3 in. There is a fireplace $6\frac{1}{4}$ ft. by 5 ft., a door and a window each $7\frac{1}{2}$ ft. by $3\frac{3}{4}$ ft. Find the number of pieces of paper required for the walls, a "piece" being 12 yds. long by 21 in. wide, a part of a piece being reckoned as a whole piece. Find also the cost at 2s. $4\frac{1}{2}$d. a piece.

$$\text{Area of 4 walls} = 2(43) \times 15\tfrac{1}{4} \text{ sq. ft.}$$

$$= \overset{43}{\cancel{86}} \times \frac{61}{\underset{2}{\cancel{4}}} \text{ sq. ft.} \qquad \begin{array}{r} 43 \\ 258 \end{array}$$

$$= \frac{2623}{2} \text{ sq. ft.}$$

$$= 1311\tfrac{1}{2} \text{ sq. ft.}$$

$$\text{Area of Fireplace} = \frac{25}{4} \times 5 \text{ sq. ft.}$$

$$= 31\tfrac{1}{4} \text{ sq. ft.}$$

$$\text{Area of a door and a window} = 2 \times \frac{15}{\cancel{2}} \times \frac{15}{4} \text{ sq. ft.}$$

$$= 56\tfrac{1}{4} \text{ sq. ft.}$$

$$\therefore \text{Area of paper} = 1311\tfrac{1}{2} - 31\tfrac{1}{4} - 56\tfrac{1}{4} \text{ sq. ft.}$$

$$= 1224 \text{ sq. ft.}$$

$$\text{Area of 1 piece} = 63 \text{ sq. ft.} \qquad 63\,|\,1224\,|\,19$$

$$\therefore \text{No. of pieces} = \frac{1224}{63} \text{ sq. ft.} \qquad \begin{array}{r} 594 \\ 27 \end{array}$$

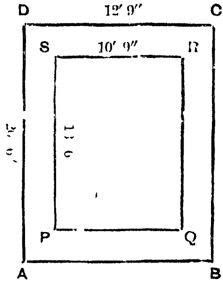
No. of pieces required, 20 pieces.

$$\therefore \text{Cost} = 2\text{s. } 4\tfrac{1}{2}\text{d.} \times 20.$$

$$= \underline{£2\ 7\text{s. } 6\text{d.}}$$

In harder questions it will be found advisable to make a figure. This adds definiteness to the question.

Example: A carpet 13 ft. 6 in. by 10 ft. 9 in. is laid on a floor 20 ft. 6 in. by 12 ft. 9 in. Find the cost of staining the rest of the floor at 3d. per sq. yd.



Let ABCD represent area of floor, and PQRS area of carpet. Then area to be stained is evidently the difference of these two areas

$$= 20\frac{1}{2} \times 12\frac{3}{4} - 13\frac{1}{2} \times 10\frac{3}{4} \text{ sq. ft.}$$

$$= \frac{41}{2} \times \frac{51}{4} - \frac{27}{2} \times \frac{43}{4} \text{ sq. ft.}$$

$$= \frac{930}{8} \text{ sq. feet.}$$

Price is 3d a sq. yard, or $\frac{1}{3}$d a sq. ft

$$\therefore \text{Cost} = \frac{930}{8} \times \frac{1}{3}\text{d.}$$

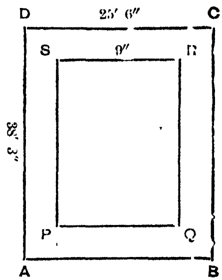
$$= \frac{310}{8}\text{d.}$$

$$= \underline{\text{3s. } 2\tfrac{3}{4}\text{d.}}$$

41
205
2091
1161
930

86
301
1161

Example: A room is 38 ft. 3 in. long and 25 ft. 6 in. wide. Find the area of a carpet for it, leaving a margin of 9 in. in width all round.



From the figure we see the length of carpet will be

= 38 ft. 3 in. − 1 ft. 6 in.

= 36 ft. 9 in.

And breadth = 24 ft.

$\therefore$ Area of carpet $= 36\frac{3}{4} \times 24$ sq. ft.

$= 147 \times 6$ sq. ft.

$= 882$ sq. ft.

$= 98$ sq. yds.

A solid has three dimensions—length, breadth, and thickness, and its volume or contents is expressed in cubic measure. A solid the shape of the ordinary brick is the usual solid

whose volume or contents has to be computed. The three measurements are multiplied together. By content is meant the space equivalent to a solid.

Example : How many bricks, each 9 in. long, $4\frac{1}{2}$ in. broad, and $2\frac{1}{2}$ in. thick, will be required to build a wall $1\frac{1}{2}$ miles long, 8 ft. high, and $2\frac{1}{2}$ ft. thick ?

$$\begin{aligned} 1 \text{ m.} &= 1760 \text{ yds.} \\ \tfrac{1}{2} \text{ m.} &= 880 \\ &\ 2640 \end{aligned}$$

$$\text{Volume of wall} = 7920 \times 8 \times \frac{5}{2} \text{ cu. ft.}$$

$$\text{Volume of 1 brick} = \frac{3}{4} \times \frac{3}{8} \times \frac{5}{24} \text{ cu. ft.}$$

$$\therefore \text{ No. of bricks} = \frac{\cancel{7920}^{2640}}{1} \times \frac{8}{1} \times \frac{\cancel{5}}{\cancel{2}} \times \frac{4}{\cancel{3}} \times \frac{8}{\cancel{3}} \times \frac{\cancel{24}^{4}}{\cancel{5}}$$

$$= 2,703,360$$

2640
7920
5280
84480
253440
168960

Example : A tank is 20 ft. 9 in. long, 15 ft. 7 in. wide, and 6 ft. 4 in. deep. How much water will it hold ?

$$\text{Contents} = 20\frac{3}{4} \times 15\frac{7}{12} \times 6\frac{1}{3} \text{ cu. ft.}$$

$$= \frac{83}{4} \times \frac{187}{12} \times \frac{19}{3} \text{ cu. ft.}$$

$$= \frac{294999}{144} \text{ cu. ft.}$$

$$= 2047\frac{131}{144} \text{ cu. ft.}$$

$$= 2047 \text{ cu. ft. } 1572 \text{ cu. in.}$$

187
1496
561
15521
31042

144 | 294899 | 2047
689
1139
131

Example : A rectangular cistern, whose length is $13\frac{3}{8}$ ft. and breadth 6 ft., contains $294\frac{1}{4}$ cu. ft. of water. What is the depth of the water ?

The content is obtained by multiplying the three measurements together; therefore one measurement is the result of dividing the content by the product of the other two.

$$\therefore \text{ Depth} = 294\frac{1}{4} \div \left(13\frac{3}{8} \times 6\right) \text{ ft.}$$

$$= \frac{\overset{11}{\cancel{1177}}}{\cancel{4}} \times \frac{\overset{2}{\cancel{8}}}{\cancel{107}} \times \frac{1}{\underset{3}{\cancel{6}}} \text{ ft.}$$

$$= 3\frac{2}{3} \text{ ft}$$

EXAMPLES XIV

1. Reduce to lbs. (i.) 2 tons 3 cwts. 3 qrs., (ii.) 5 cwts. 1 qr. 16 lbs., (iii.) 15 cwts. 2 qrs. 19 lbs., (iv.) 14 cwts., (v.) 1 ton, (vi.) 3 cwts. 12 lbs. (vii.) 3 qrs. 14 lbs., (viii.) 13 cwts. 2 qrs. 16 lbs.

2. Express each of the quantities in question 1 in cwts.

3. Reduce to inches (i.) 44 yds. 1 ft. 9 in., (ii.) 104 yds. 10½ in. (iii.) 88 yds. 2 ft. 10 in., (iv.) 2 miles, (v.) 8 yds. 1 ft. 4 in., (vi.) 13 yds. 1 ft. 7 in., (vii.) 5 chains 20 yds., (viii.) 21 miles 427 yds. 1 ft. 5 in.

4. Express each of the quantities in question 3 in yds

5. Reduce to pence: £45 15s., £18 18s. 8d., £3 2s. 6d., £1,504 11s. 10½d., £89 1s. 4d., £17 5s. 3¾d., 9s. 10d., 50,641 halfpence, £321 10s. 6d.

6. Express in £ s. d: 4,031d., 1,815d., 365d., 404,123 halfpence, 99 sh., $314\frac{7}{8}$ sh., 500d., $83\frac{5}{24}$ sh., 6,442d., 30,456 farthings.

7. Express (i.) in £, (ii.) in sh., (iii.) in d., each of the following:— £3 11s. 3d., £11 16s. 3d., 5s. 6d., £6 9s. 4d., £41 12s., £1,620 7s. 4½d., £116 17s. 6d., £11 1s. 10d., £846 3s. 4½d.

8. A load of coal weighs 7 tons 14 cwts. 3 qrs. 15 lbs., and the cart weighs 2 tons 5 cwts. 1 qr. 17 lbs. Find weight of the coal.

9. A man buys 6 cwts. 1 qr. 15 lbs of sugar at 3½d. a lb., and sells it at 4d. a lb. How much profit does he make?

10. Find area of a rectangle 53 ft. 9 ins. long and 17 ft. 7 in. wide.

11. Find in sq. ft. the following areas:—19 ft. by 7 ft., 105 ft. by 19 ft., 304 ft. by 25 ft., 8·21 ft. by 1·25 ft., 17 ft. by 9 in., 2 in. by 3 in., 15 yds. by 13 ft., 7 yds. by 7¼ in.

12. A field measures 71 yds. by 22 yds. What fraction of an acre is its area?

13. The area of a rectangle is 225 sq. yds., and its length is 15 yds. How wide is it? If its length had been 15 ft. how wide would it be?

14. Multiply 3 bus. 2 pks. 1 gal. by 7.

15. How many seconds are there in 4 days 3 hrs. 12 min.?

16. How many sq. yds. of carpet will cover the floor of a room $22\frac{1}{2}$ ft. long and 17 feet wide? If the width of the carpet is 27 inches. how many yds. of carpet will be needed?

17. How many sq ft are there in a passage $17\frac{3}{4}$ ft. long and $3\frac{1}{2}$ ft. wide?

18. How many cubic feet of air are there in a room 18 ft. 6 in. long, 12 ft. wide, and 10 ft. high?

19. How many acres are there in a field 300 yds. long and 180 yds. wide? Give the result to nearest acre.

20. A tank 6 ft. long and 4 ft. wide contains 30 cubic feet of water. What is the depth of the water?

21. A barrel of ale containing 36 gallons cost £2 2s. How many half-pint glasses can be drawn from it, and what will be the whole profit by retailing the ale at 2d. a half-pint glass?

22. A room is 25 ft. 6 in. long and 19 ft. broad. The floor is stained to a distance of 9 inches from each wall, and the rest is covered with a carpet. Find the total expense if the staining costs 1s. 6d. per sq. yd. and the price of the carpet, 30 ins. wide, is 4s. per yd.

23. If a cubic foot of lead weighs 704 lbs., find the weight of sheet lead, $\frac{1}{16}$th of an inch thick, covering a balcony 12 ft long and 2 ft. 9 in. broad.

24. The breadth of a rectangle is 2·6 inches. and its area is 11·7 sq. inches. How long is it?

25. Reduce 2 miles 6 chains 17 yds. 2 ft. to inches.

26. What length of string will be required to go both ways round a box, with a lid, 3 ft. 6 in. long, 2 ft. 9 in. wide, 1 ft. 9 in. high, the string to cross the lid both times?

27. The edges of a brick are 1·3 ft. 7·8 inches and ·12 yd. Find its volume in cubic feet.

28. A room 18 ft. long and 14 ft wide is flooded with water $1\frac{1}{2}$ in. deep. Calculate the weight of water if a cu. yd. of water weighs 15 cwt.

29. Find the area of the walls of a room 23 ft. 8 in. long, 18 ft. 4 in. wide, and 12 ft. high; and the expense of covering them with paper 3 ft. 6 in. wide costing 4s. 6d. per dozen yards.

30. Find to the nearest inch the length of paper which would be used in papering the four walls of a room 21 ft. long by 13 ft. broad by 8 ft. 6 in. high, if no allowance were made for windows, &c., the breadth of the paper being 20½ inches.

31. What will be the cost of laying a hall 84 ft. by 45 ft. with linoleum at 3s. 6d. a sq. yd., allowing 5 per cent. extra for waste.

32. How many yds. of paper 2 ft. wide will be required for the walls of a room 18 ft. long, 16½ ft. broad, and 12 ft. high?

• 33. The dimensions of a class-room are 25·5 ft. by 16·4 ft. by 11·2 ft. Find to the nearest cubic inch how much space there is to each boy in a class of 29.

34. What is the cost of painting at 2s. 6d. a sq. yd. the walls of a room 20¼ ft. long, 18½ ft. broad, and 10 ft. high, containing two windows each 7 ft. by 4 ft.?

35. The height of a room is 11 ft. 9 in., length 18 ft., breadth 13 ft. 6 in. How many pieces of paper 21 inches wide and 12 yds. long will it take to paper the walls? There is a fireplace 5 ft. high and 5 ft. broad, a door and a window each 7 ft. by 4 ft. Find also the cost if the paper is 1s. 5½d. per piece.

36. A plot of land 16 ft. × 22 ft. yielded 17 stones of potatoes. How many tons is this to the acre?

37. Find the number of bricks 9 in. × 4½ in. × 3 in. required to build a wall 1,000 yds. long, 1 ft. 3 in. wide, and 9 ft. high, 12½ per cent. of the space being occupied by the mortar.

CHAPTER XV

METRIC OR DECIMAL SYSTEM

In the previous chapter were given the principal tables of weights and measures as used in England. In many other countries another system has been adopted, known as the Metric. It was made compulsory in France in 1802, and is now the system established in most other countries. The chief points aimed at in this system are two—first, that all tables should have the same equivalents, i.e., one definite unit is taken for each set of measures, and all the others are powers of tens of this unit; second, that all the various units of measure should be derived from a fundamental unit, the metre.

All the tables are constructed in the same way, namely, on a decimal basis, as follows:—For multiples of the unit the Greek numerals deca $=10$, hecto $=100$, kilo $=1{,}000$, are prefixed, and for sub-multiples the Latin numerals, deci $=\frac{1}{10}$, centi $=\frac{1}{100}$, milli $=\frac{1}{1000}$, are similarly used. So that once these prefixes are learned, all tables are known. For example, the unit of length is the *metre*, and we have

Table of Length.

Kilometre	$=10$ hectometres.
Hectometre	$=10$ decametres.
Decametre	$=10$ metres.
Metre	$=10$ decimetres.
Decimetre	$=10$ centimetres.
Centimetre	$=10$ millimetres.

Another way of learning the table is by arranging it horizontally, thus:—

1,000	100	10		$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
Kilo-	Hecto-	Deca-	*Metre*	deci-	centi-	milli-

It will be seen that reduction, i.e., changing from one denomination to another, is effected by a simple moving of the decimal point. It is only necessary to look at the above horizontal table to know how many places to move. Example: Express 375 centimetres in metres. Metre is two places higher, or to the left. $\therefore$ 375 cm. = 3·75 metres.

Example: 23·74 metres = 23,740 mm., or = 2,374 cm., or = ·02374 km.

For commercial purposes the only measures of length are metre, centimetre, and kilometre. Where we use yds., ft., and inches, the metre and centimetre are used. The kilometre is for larger distances where we use miles.

Square Measure.

For measuring areas the sq. metre, &c., are often used, but the student must notice that the equivalents will go by 100, not 10. Thus 1 metre = 100 centimetres, but 1 sq. metre = 10,000 sq. centimetres, or $(100)^2$ sq. cm.

Land is generally measured in ares, and the only derivatives in use are the hectare and centiare. The hectare would be used where we use acres. 1 are = 1 sq. decametre.

Cubic Measure.

For volume the cubic metre, &c., are used, but the table will, of course, advance by 1,000 each time, i.e., 10^3.

The cubic metre is called a stere, and is used in the timber trade.

Liquids are measured by litres, &c.

A litre is a cubic decimetre, the only common measures being the litre and the hectolitre.

Weight.

The unit of weight is called the gram (sometimes spelled gramme). The only weight common in commerce is the kilogram.

The student should remember that the

Unit of length is the metre.

Unit of capacity is the litre.

Unit of weight is the gram.

The names of the measures are abbreviated to m. for metre, l. for litre, g. for gram, Km., Hm Dm. for kilometres, hectometres, dekametres; dm, cm., mm., for decimetres, centimetres, millimetres. In the other denominations similar contractions are used. It will be evident that all the working will be in decimals.

Addition and Subtraction.—Arrange all the quantities in one denomination, and work as in decimals.

Example: Add together 13 m. 8 cm., 17 m., 5 cm., 75 cm., 3 km., 4 mm.

13·08	metres
17 05	,,
·75	,,
3000·	,,
·004	,,
3030·884	metres

Multiplication as in decimals. In measuring lengths it will generally be sufficient to work correct to two decimals in metres, i.e., to the nearest centimetre.

Example: Multiply 532·75 m. by ·123.

Changing the point in multiplier to give a unit figure, we have

m	
53·275	× 1·23
10·655	
1·598	
65·53	metres.

Division. Example: A rod of iron weighing 15 kg. is cut into 40 equal pieces. Find weight of each in grams.

$$40 \,|\, \underline{15{,}000}$$

375 grams.

Example: A man rides a bicycle at the rate of 5·75 m. a second. How long will he take to ride 100 km.?

$$\text{No. of seconds} = \frac{100{,}000}{5 \cdot 75}$$

$$= 17391 \text{ secs. nearly}$$

Time = 4 hrs. 49 min. 51 secs.

```
5·75|100000 |17391
    | 4250  |
    |  2250 |
    |   5250|
    |    750|
    |    175|

60|17391
60|  289,,51
  |    4,,49
```

The various units are connected together by the following relations :—

One litre = 1 cu. dm.

One gram is the weight of 1 cc. of water under certain conditions, viz., of pure water at 4° C., and at pressure 760 mm.

Example: A vessel measures 5 m. by 2·5 m. by 30 cm. How many litres of water will it hold?

Volume	=	5 × 2·5 × ·3 cubic metres
	=	3·75 cu. metres
1 cu. m.	=	1,000 cu. dm.
	=	1,000 litres.
∴ Capacity	=	3,750 litres

Decimal Coinage.

Many countries have also a decimal system of coinage, though the coin of unit value varies both in name and value. Thus in France the unit of value is called the franc, in Germany the mark. The monetary unit is in all these countries subdivided into 100 smaller units, as per following table :—

France: 1 franc = 100 centimes. In accounts sums of money are written 312 fr. 15., it being understood that the 15 is 15 centimes.

Germany: 1 mark = 100 pfennige.

In the United States the unit of value is the dollar, the sign for which is \$, and it is sub-divided into 100 cents.

Calculation of prices is performed by decimal multiplication.

Find price of 37 m. 45 cm. of cloth at 2 frs. 35 per metre.

$$\text{Price} = \overset{\text{frs.}}{2\cdot35} \times 37\cdot45 = \overset{\text{fr.}}{88\cdot01}$$

37·45	× 2·35
74·90	
11·235	
1·873	
88·01	

As the centime is a small value the answer need only be correct to 2 decimals. Quantities in the decimal systems may be converted into the English system and *vice-versa*. The subject will be treated more fully in a further chapter. For simple conversions the student may refer to the accompanying table of equivalents, taken from an Order in Council of May, 1898 :—

Imperial Equivalents			Metric Equivalents.		
1 metre	=	39·370113 ins.	1 yd.	=	·914399 metre
1 hectare	=	2·4711 acres	1 grain	=	·0648 grams
1 gram	=	15·432 grains	1 lb.	=	·45359243 kg
1 kilogram	=	2·2046223 lbs.	1 gallon	=	4·5459631 litres
1 litre	=	1·7598 pint			

The following rough equivalents might also be learned.

A metre $=39$ inches nearly.
A kilometre $=5$ furlongs.
A centimetre $=\frac{2}{5}$ of an inch.
A hectare $=2\frac{1}{2}$ acres.
A litre $=1\frac{3}{4}$ pints.
A kilogram $=2\frac{1}{5}$ lbs. av.

The values of the principal coins are approximately

$9\frac{3}{4}$d. for 1 franc.
$11\frac{3}{4}$d. for 1 mark.
50d. for 1$ (U.S.).

Example: Convert $125\frac{3}{4}$ yds. into metres.

$$125\tfrac{3}{4} \text{ yds.} = 125{\cdot}75 \times {\cdot}914399 \text{ metres.}$$
$$= 114{\cdot}98 \text{ metres.}$$

$$\begin{array}{r} 12{\cdot}575 \times 9{\cdot}14399 \\ \hline 113{\cdot}175 \\ 1{\cdot}258 \\ {\cdot}503 \\ 38 \\ 11 \\ 1 \\ \hline 114{\cdot}98 \end{array}$$

It is generally sufficient to work to 2 decimals, i.e., to nearest centimetre.

Example: Express in francs and centimes £41 12s. 6d., being given that £1 = 25·22 francs.

$$£41\ 12s.\ 6d. = £41{\cdot}625$$

$$\begin{array}{rl} 41{\cdot}625 \times 25{\cdot}22 & \\ \hline 1040{\cdot}625 & (= 25 \text{ times}) \\ 8{\cdot}325 & (= {\cdot}2 \text{ times}) \\ {\cdot}832 & (= {\cdot}02 \text{ times}) \\ \hline 1049{\cdot}78 & \text{or } 1049 \text{ frs. } 78 \text{ c.} \end{array}$$

Example: Express in £ s. d. to nearest farthing 375 francs, if £1 = 25·22 frs.

$$\begin{array}{l} 25{\cdot}22)375{\cdot}00(14{\cdot}869 \\ \quad\quad 122\ 80 \\ \quad\quad\ \ 21\ 920 \\ \quad\quad\quad 1\ 744 \\ \quad\quad\quad\quad 231 \\ \quad\quad\quad\quad\quad 4 \end{array}$$

$$\text{Answer} = \underline{£14\ 17s.\ 4\tfrac{3}{4}d.}$$

Example: I buy 40 metres of velvet at 12 fr. 60 a metre. What is the cost approximately in English money?

$$\begin{aligned} \text{Cost} &= 12{\cdot}60 \times 40 \\ &= 504{\cdot} \text{ frs.} \\ &= £19{\cdot}984 \\ &= £19\ 19s.\ 8d. \text{ nearly} \end{aligned}$$

$$\begin{array}{l} 25{\cdot}22)504{\cdot}0(19{\cdot}984 \\ \quad\quad 251\ 80 \\ \quad\quad 248\ 20 \\ \quad\quad\ \ 21\ 22 \\ \quad\quad\quad 1\ 04 \end{array}$$

The coinage of India is not on a decimal basis. The unit of monetary value is the rupee. 100,000 units is called a *lac*, and the method of writing large numbers is as follows:—125,000 rupees would be written 1,25,000, and read as one lac 25 thousand. The rupee is sub-divided into 16 annas, and 1 anna into 12 pie.

The rupee is equivalent to 1s. 4d. One hundred lacs is called a crore, so that Rs. 4,00,00,000 is read as 4 crores rupees. Rs. 2,21,25,000 is read as 2 crores 21 lacs 25 thousand rupees.

EXAMPLES XV

1. The weights of four parcels are 5·37 kg., 1·8 kg., 3·85 kg., and 2·06 kg. respectively. What is their combined weight?

2. A litre is 1·76 pints. How many pints are there in 3·5 litres?

3. How many times can a jug which holds a litre be filled from a cask containing 2·08 hectolitres?

4. Find the value of 725 grams of gold at £187 per kilogram.

5. A nugget weighed a kilogram, and 0·36 of it was gold. How many grams of gold did it contain?

6. If there is 19 per cent of water in a mixture, how many centilitres of water are there in 2 litres of the mixture?

7. If the price of an article in England is 3 sh., find to the nearest 5 cents what the price of it would be in American money ($1 = 50d.).

8. How long will a pipe, which supplies 20 litres of water per minute, take to fill a tank which holds 5 hectolitres?

9. A tramline 3 Km long had a length of 85 metres laid as single line, and the rest was double line. What length of the line was double?

10. Taking a franc as 9½d., give the value of 15 francs in shillings to the nearest shilling.

11. If a metre be taken as 3·28 ft., how many sq. ft. will there be in a sq. metre?

12. A cistern is 1·15 metres in length and 0·65 metres wide. Find, to the nearest centimetre, the depth of water in it when it holds 400 litres.

13. A kilogram is 2·2 lbs. What is the price of a pound of tobacco if a kilogram cost 12 sh.?

14. In a kilogram of brass there are 250 grams of zinc. What percentage of the brass is zinc ?

15. How many litres are there in 3·7 hectolitres ?

16. What depth of water in centimetres are there in a tank 3 metres long and 2 metres wide, when it holds 3 cubic metres of water ?

17. From 53 hectolitres 46 litres of wine, how many bottles can be filled, each containing 1 litre ?

18. How many lbs. (to nearest lb.) are there in 545 Kg ?

19. A railway is being laid down at the rate of 0·24 Km. a day. How long will it take to lay 100 Km. ?

20. How many cubic metres of air space are there in a room 10 metres long, 8 m. 50 cm. broad, 7 m 20 cm. high ?

21. An inch is 2·54 cm. How many inches are there in 12 cm. (to nearest inch) ?

22 If a train runs 73·5 Km. in an hour, how long will it take from London to Edinburgh, 660 Km. (to nearest hour) ?

23. How many sq. metres are there in a sheet of paper 1·36 metres long and 0·75 metres wide ?

24. Convert the following lengths into yards :—945 metres, 401 metres, 99·75 metres, 12.025 metres, 660 centimetres, 3 Km. (answers correct to 3 decimals of a yard).

25. Convert the following lengths into metres (correct to centimetres) :—700 yds., 88 yds., 125 yds., 27 yds., 67·5 yds., 80 yds. 9 in., 101 yds. 2 ft., 24½ yds., 44¼ yds., 37¾ yds., 8⅛ yds.

26. Convert the following weights to lbs. (correct to nearest lb.) :— 30 Kg., 805 Kg., 93·75 Kg., 13,478 grams, ½ Kg., 88½ Kg., 469 Kg.

27. How many miles are there in 250 Km. ?

28. How many steps, each 65 cm. long, does a soldier take in marching 273 Km. ?

29. A piece of wire 150 cm. long weighs 1,320 gm. What length of wire will weigh a kilogram ?

30. If I buy 4½ litres of petrol at 2 francs for 5 litres, how much do I spend ?

31. A litre of wine is worth ·45 francs. How many litres would one get for 146 fr. 25 c. ?

L

32. A man bought some wine at 32 fr. 50 the hectolitre, some at 36 fr. 60, and some at 40 fr. 80. If he mixes equal quantities of these so that the value of the mixture is 1,593 fr. 55, how many decalitres of each kind did he buy?

33. Find the area of a rectangle 1 m. 60 cm. long and 75 cm. wide.

34. How many sq. metres are there in the area of the two faces of a wall 16 m. 50 cm. long, 6 m. 30 cm. high, in which there are 3 windows each of height 2 m. 10 cm., and width 1 m. 30 cm.?

35. Find the price of 45·68 grams of gold at 3,442 frs. 15 the Kg.

36. Find the simple interest on 3,431 frs. 60 for 217 days at $1\frac{1}{2}$ per cent.

37. In one year France produced 83,632,391 hectolitres of wine. Give to the nearest integer the equivalent in millions of gallons. Take 1 litre = 1·761 pints.

38. Taking £1 = 25·22 frs., convert into francs and centimes the following amounts:—£30, £907 15s., £8 12s. 9d., 17s. $10\frac{1}{2}$d., 12s. 6d., £23 11s. 5d., £1,000. Also convert into £ s. d. 20,000 frs., 183 f. 50, 4,021 f. 75, 633 f. 35, 45,275 frs.

39. Read the following:—3,23,10,000 rupees, 65,00,000 rupees

40. Find the equivalent in English money of 76 rupees 10 annas 5 pie.

41. A gentleman employed in India remits home a lac of rupees How much English money is that?

ANSWERS

CHAPTER I

1. 54,512

2. £1.

3. 8,769

4. 231,038

5. 28·25d

6. 10,080,507

7. 22·71

8. £48 6s. 3d

9. 46·08 kg

10. 16,852

11. Columns 36,998, 34,200, 45,758, 470,042, 459,345 Rows: 128,067, 157,930, 98,775, 36,996, 13,706, 57,773, 65,103, 31,162, 79,941, 22,033, 55,750, 50,358, 11,158, 22,066, 105,452, 90,013, 20,060. Total, 1,046,343

12. Totals £30,891 8s 4d., £49,971 18s. 5d Omission of £19,080 10s. 1d in Dr column

13. £12,550 14s 2d. inc., £2,981 12s. 5d. inc., £10,512 12s. 0d inc., £5,129 13s 3d inc., £4,572 13s 5d inc., £358 3s 1d. inc., £1,192 0s 9d. dec., £1,641 9s 9d dec., £778 2s 3d dec., £427 16s 2d inc., £195 14s 9d. dec., £140 9s. 8d inc., £329 19s 1d dec., £256 1s 1d. dec., £343 2s. 6d inc. Totals £142,116 14s 5d., £103,215 5s 11d

CHAPTER II

1. 76,703,102,468, 1,591,219, 1,150,398, 14,205,361, 7,178,169,684, 299,425, 25,551,675

2. 8,353,125, 37,545,758, 9,851,310 (800 − 2 times), 5,050,582,116, 431,816, 71,753, 868,900, 20,085, 61 875, 10,154,250, 843·75 (1·25 × 8 − 10), 434·375.

3. 3,476·5, 4·59, 10·101, ·26783, ·89341, ·0023, 9,684·5, 26·125, 1·6225, 84,375, 15,625, 156·25, 4·16625.

4. £11 12s 6d., £1,721 1s., £7,474 2s. 7d., £6,650 6s. 6d., £1,065 13s 0$\frac{3}{4}$d., £25,201 16s. 11d.

5. £4 (1s. 4d = £$\frac{1}{15}$).

6. 341 miles.

7. 11 cwt. 82 lbs.

8. £53 6s.

9. 122 tons 12 cwt. 2 qrs.

10. £1 16s. 8d.

11. £8 15s. 6d. (1st Jan , 1904, was a Friday).

12. 12·8 kg.

13. 1 68834, 341 83795, 35·70495807, ·003282751, ·181447, 66,769·92, 658·8697824.

14 £3,712 18s. 4d., £2,542 18s 6d , £570 8s 7d , £434 14s. 1d , £1,195 5s 9d. ; Total, £8,456 5s. 3d.

15 £707,430 19s

CHAPTER III

1. 49,102, 1,051,546, 8 281, 2837·25, 1 004166 . . . ., 0 031233 . . ., 7,134, 5,115$\frac{4}{13}$, £39 0s. 6d , £7 19s 7d , £3 17s 3$\frac{3}{4}$d , £5 7s 0$\frac{7}{8}$d

2 5,844$\frac{141}{165}$, 1,172$\frac{1}{2}\frac{6}{14}$, 43,725, 1·457, 1,935, 0·0993, 0·0009455, £10 19s. 11$\frac{7}{8}$d , 7$\frac{1}{4}$d , £197 0s. 1d.

3 8s.

4. 1s 0$\frac{1}{2}$d.

5 307, 2s 7$\frac{1}{2}$d.

6. 27 lbs.

7 144 books

8 139.

CHAPTER IV

1 3 458, 18 98, 60,634·8, 9·716, ·0492

2. 108·506.

3. 31·87642.

4. £5 308, £3·670, £169·106, £0 409, £17·212, £1·965, £0·125, £0·925, £0·193, £0·387, £49 583, £2·082.

5. £2·154166 . . ., £15·3125, £8·8604166 . . ., £9·8145833 . . ., £14·35520833 . . , £1·420833 . ., £2·815625, £7506·875, £834·525, £7·725, £8,612·1833 . ., £99·779166 . . ., £6·7375, £304·59375, £0·13020833 . , £0·3333 . . . ., £0·7104166 . ., £0·2875, £0·345833 . ., £0·666 ., £245·60625, £0·01145833 . ., £0 015104166 . . (3$\frac{5}{8}$d. = 14$\frac{1}{2}$f. = £·0145, 3·625 ÷ 6,000 = ·000604166 . . .)

6. £6 2s 3d., £7 14s 3¾d., £3 4s. 10¾d., 5s. 5½d., £3,456 2s 6d., £81 18s 3d., £1 2s 2¾d., £16 18s. 9d., £37 6s 5¼d., £82 16s 6¾d., £9 2s 11¾d., 4s. 1¼d., 3½d., 2s. 6½d., 2d., 2¾d., 10s., 4s. 9¾d., 12s. 1½d., 14s. 5¾d., 17s. 0½d.

7. 6·411, ·1230, 39·4431, 528·7, ·002, 7·4480, 1·1868, 70·118, 141·615, ·1471, 8·5649, 100·770, 120·348, 7·539, 101·731, 3,023·738, 30·3992.

8. 54·92, 267, 1·643, 13·35, 10·989, 28,838·693, 2·5991, ·000005, 100·0000, ·16, 121,932,631

9. 25·17, 60·5002, ·005, 110·882, ·014, 2·421, ·74, 42·33, 9.

10. 1d., £2 12s. 3d., £1 0s. 4d., £56 9s. 7d., 14s 3d., 1s. 11d., 15s.

11. 2s 8d

12. 8s 1½d

13. 1s. 3d.

14. £613 1s 5¼d.

15.

£1,475·6666	
44·2700	
1,519·9366	
45·5981	
1,565·535	£1,565 10s. 9d.

16.

£4,750
332·5
4,417·5
309·225
4,108·275 = £4,108 5s 6d.

17. £16 5s 10d., £65 18s. 11½d., £8 2s. 10½d., 2s. 7¼d.

CHAPTER V

1. £162 18s. 6d.
2. £3,452 13s. 10d.
3. £289 13s. 4d.
4. £351 9s. 4½d.
5. £707 1s. 0¼d
6. £152 15s.
7. £12 14s. 7¾d.
8. £584 4s. 3d.
9. £62 10s.

10 £1,400 13s

11. £7 11s. 10d.

12. £646.

13. £6 6s. 6¾d.

14 £109 5s. 4d

15 £425 5s. 5¾d

16 £533 18s

17. £337 2s 6d

18 £549 13s 4d

19 £83 6s. 8d

20 £689 10s

21 £59 8s

22 £51 17s. 10½d

23 £3,494 19s 7½d

24. £56 12s. 7¾d

25. £331 14s 2d

26. 6s 10½d., £10 7s 8½d , £1 0s. 4d , £4 19s 2¼d , £343 11s. 0½d

27. £18 4s , £2 2s., £1 2s 2d , £265 8s 4d , £51 2s 8d.

28. 7s. 6d., 10s. 3d., £1 6s. 3d , £2 2s , £3 10s

29 £14 8s. 11½d

30. £19 3s. 2d.

31. £62 15s. 5d.

32. £140 2s. 7½d.

33. £14 18s 3¼d

34. £79 15s. 10d.

35.

£	s.	d.
	15	11¼
	15	2¼
1	15	6
	12	4½
	3	9
4	2	9

36. £323 17s. 11½d.

37. £112 9s. 5d.

38. £6 2s. 8d.

39. £2 13s. 4d.

40. ¾d.

41. £5 1s. 7d., 2⅞d.

CHAPTER VI

1. 8 horses.
2. 11 min.
3. £192.
4. £2 5s.
5. 9 men.
6. £874 5s
7. £59.
8. 12 days
9. £20 19s 4d
10. £2 5s. 9d
11. 4 hrs 48 min
12. £119 7s 6d
13. £64.
14. £16 10s
15. $\frac{51}{160}$
16. £42 11s. 5d.
17. 209 min.
18. 22 men.
19. $£690 \times \frac{24}{23} \times \frac{19}{20} = £684.$
20. 14 oz.
21. 360 men
22. 10s 10d
23. £3,750
24. 2s. $8\frac{1}{2}$d., 10s
25. 2s. $9\frac{1}{4}$d.
26. £2,500.
27. £1,653 6s. 2d.
28. $£236 \times \frac{60}{59} \times \frac{5}{4} = £300.$
29. £300 16s.
30. 75 burners

CHAPTER VII

1 $\frac{1}{10}$, $\frac{6}{25}$, [illegible], 1, $\frac{7}{20}$, $\frac{1}{100}$, $\frac{1}{200}$, $\frac{1}{800}$, [illegible], [illegible], [illegible].

2 50%, 25%, 12½%, 6¼%, 14$\frac{2}{7}$%, 11$\frac{1}{9}$%, 66⅔%, 75%, 80%, 93¾%, 86[illegible]%, 2%, 21%, 22½%, 1¼%, 4⅝%, 2¼%

3 £5 10s, £2 17s 8d, 10½d, £51 1s 5d, 5s, 6s 10½d, 8s 1d, 21, 3 57, 6,641 37, £1 19s 1d, £[illegible]0 5s 5¼d, £15 1s 6d, £762 4s 10½d, £13 6s 8d

4 £61 16s, £807 8s 7¼d, 12,533 57, 11,623 3125, 979 02, £6,701 10s 6d, £35,975 9s, 36,421 11, 1,711 29, 899 1, £780 4s 2d, £6,578 16s 5d, £2,424 10s. 8d, £9,159 7s 5d, £332,737 1s 3d, £530 2s 9d

5 8,900

6 £14 0s 1½d

7. 25%.

8 97 .

9 £3 15s

10. 2%

11 76%, 10%, 11%

12 1,120 grams

13 25 23 %, 30 77 %

14. 1,722.

15 8%.

16 84% in 1878, 7% in 1900, 87% dec, 11,417,900% inc.

17 £1,548

18 £143,960.

19 62 9%, 63 5%, 63 4%.

20. £163 14s 8d

21. £2,000
      80
   1,920
    192
     96
   1,632
    408
   1,224

22. £714 18s

23. 193,650

24. 4% dec., $66\frac{2}{3}$% inc

25. £90. $\left[\frac{105}{100}\right.$ of £10 = £10·5, £15 − 10·5 = £4·5, which must equal increase of earnings; but difference of earnings fraction $= \frac{110}{100} - \frac{105}{100} = \frac{5}{100}$. Earnings $=$ £4·5 $\times \left.\frac{100}{5}\right]$

CHAPTER VIII

1. £31 5s., £85, £19 10s. 11d., £9 8s. $1\frac{1}{2}$d., £17 2s. 3d., £35 5s. 5d., £15 17s. 3d., £3 14s. 9d., £51 11s. 10d.

2. £377 8s. 4d., £262 3s. 9d., £136 10s., £1,710 14s., £876 1s. 7d., £941 5s.

3. £1 15s., £18 4s. 7d., £292 13s. 9d., 7s. $4\frac{1}{2}$d.

4. £373 19s. 7d.

5. £7,347 2s. 10d.

6. £4,166 13s. 4d.

7. £1,151 17s. 6d.

8. £21 7s. 6d., £10 18s. 8d., £30 1s. 6d.

9. $16\frac{2}{3}$%

10. 2·89%

11. £14 0s. 6d.

12. £500

13. $\frac{11}{130}$.

14. £12,326 14s.

15. £1 18s. 11d.

16. £6 8s. 11d.

CHAPTER IX

1. 25% profit, $9\frac{1}{11}$% loss, $66\frac{2}{3}$% profit, $28\frac{4}{7}$% loss, 28% profit, $21\frac{7}{8}$% loss.

2. 5s. 9d., $9\frac{3}{8}$d., £1 13s. 7d., 4s. $1\frac{1}{4}$d., £1 5s. $2\frac{1}{2}$d.

3. 4s. 3d., $5\frac{5}{8}$d., £1 8s. 5d., 2s. $0\frac{1}{4}$d., 10s. $9\frac{3}{4}$d.

4. £7 2s. 10d., 8s. 4d., £5 10s. 2d., 7s. $2\frac{1}{2}$d.

5. £25 16s. 8d., £1 1s. $10\frac{1}{2}$d., 1s. 3d., £5 5s., £1 11s. 8d.

6. $48\frac{1}{13}$%.

7. 44%.

8. 1s. 1d

9. Cost price 15s., selling price 17s. 6d.

10. $22\frac{2}{9}\%$.

11. 56s

12. $43\frac{3}{4}\%$

13. £468 15s., $6\frac{2}{3}\%$.

14. 10d., $4\frac{1}{6}\%$ profit

15. £25 10s

16. 5s. 11d., $7\frac{2}{7}$d

17. $3\frac{1}{4}$d., nearly

18. $31\frac{11}{19}\%$

Cost Price ← List Price

$\frac{95}{100}$

$\frac{110}{100}$

less discount

$\frac{88}{100}$

Net Price.

$\therefore$ fraction $= \dfrac{110}{100} \times \dfrac{100}{88} \times \dfrac{100}{95}$

19. £128.

20. $29\frac{1}{6}\%$ (buys at 90, sells at $120 \times \frac{31}{32}$).

21. C.P. fraction $= \dfrac{75}{100} \times \dfrac{95}{100} = \dfrac{171}{240}$

S.P. fraction $\dfrac{5}{6} = \dfrac{200}{240}$

$\therefore$ profit rate $\dfrac{29}{171} = 16\frac{164}{171}\%$

22. C.P. per article $= £\left(40 + 10 + \dfrac{350}{52}\right) \times \dfrac{1}{100} = 11\text{s. } 4\frac{1}{4}\text{d.}$

Retail price per article $= \dfrac{1770}{13}\text{d.} \times \dfrac{130}{100} \times \dfrac{100}{90} \times \dfrac{100}{88}$

$= 18\text{s. } 7\frac{1}{2}\text{d.}$

Annual cost $= £50 \times 52 + 350 = £2,950.$

Annual sales $= £\cdot93125 \times 5,200 = £4,842\cdot5.$

Cash $= £4,842\cdot5 \times \dfrac{88}{100} = £4,260\cdot960.$

$\therefore$ Amount received $= £4,260\cdot96 \times \dfrac{99}{100} = £4,218.$

$\therefore$ Total profits $= £1,268.$

23 6s. 6d

24 1st C. P. per lb $= 64d \times \frac{100}{128} = 50d.$,

2nd C. P. per lb $= 44d.$

$\therefore$ Profit $= \frac{12}{44}$ or $27\frac{3}{11}\%$.

25 £1 6s 8d

26 $21\frac{7}{8}\%$

27 336 lbs

28 $26\frac{6}{15}\%$

29 £9 15s 10d.

30 $24\frac{1}{2}\%$

31 £152 1s 8d.

32 560 lbs

33 50%

34 £1 2s 6d

35 1d each

36 £24

37 $16\frac{2}{3}\%$

38 First by 1s.

39. £1 0s. $7\frac{1}{2}$d.

40. $9\frac{1}{11}\%$

41 £5,020.

42. $9\frac{1}{11}\%$

43. 2s. 9d

CHAPTER X

1 £22 10s , £4 13s. 9d , £151 3s , £21 18s.. £208 1s. 4d

2 £2 2s , £4 1s , £2 7s. 10d , £9 2s., £18 17s. 1d , £18 10s. 4d., 16s 6d., £10 17s. $1\frac{1}{2}$d , £8 15s , £312 10s.

3. £1 10s. 4d., £2 8s 1d , £5 0s. 2d , £8 12s 6d , £14 16s. 2d., £43 16s., £10 4s. 6d., £128 9s. 3d., £2 19s. 1d., £257 3s. 4d.

4 £1,085 8s. 9d.

5 £4 12s. 7d

6. £2 1s. 1d.

7. £113 13s.

8. £289 19s 6d.

9. £4 14s. 8d.

10. £3 10s. 6d.

11. £33 2s. 7d.

12. £15 15s. 4d.

13.

			Balances			Days	Products.
Jan	1		£250	6	3	4	1000
,,	4		245	6	3	21	5145
,,	25		255	18	9	2	512
,,	27		248	8	9	11	2728
Feb.	7		273	14	9	25	6850
March	3		257	18	11	56	14448
April	28		331	5	7	12	3972
May	10		314	5	7	35	10990
June	14		239	0	7	16	3824
,,	30	Int.	3	7	9		·4946|9 × 5
July	1	Bal	£235	12	10		2·4735
							8245
							825
							82
							3 389

14 £74 10s. 1d

CHAPTER XI

1. £1,061 4s. 2d
2. £512 9s. 1d.
3. £2,224 1s 9d
4. £3,651 11s 7d.
5. £621 13s. 5d.
6. £14,060 8s.
7. £787 0s. 1d.
8. £395 18s 11d.
9. £12,762 16s. 4d.
10. £4,194 2s. 10d.
11. £876 19s. 8d.
12. £16 5s. 10d.
13. £7,132 5s. 9d
14. £1,529 16s. 4d.
15. £1,137 18s. 3d.
16. £21,275 3s. 1d.
17. £611 0s. 3d.
18. £657 19s. 1d.

The Interest is obtained by subtracting principals from answers 1–18.

19. £25 6s. 9d.

20. £12 12s.
21. £1,324 18s.
22 £646 10s 5d
23. £5 12s 5d
24. £179 11s 9d.
25. 1,093 per 1,000.
26 £4,255 15s. 1d.
27. 5 0945 %
28 £56 11s. 5d
29. 324,163
30 £3,936 12s
31.

	£
	160,000
+	8,000
	168,000
−	45,121 8917
	122,878 1083
+	6,143 9054
	129,022 0137
−	45,121 8917
	83,900 1220
+	4,195 0061
	88,095 1285
−	45,121 8917
	42,973 2364
+	2,148 6618
	45,121 8982
	45,121 8917

CHAPTER XII

1 $3\frac{1}{8}\%$, 6%, $12\frac{1}{2}\%$, 4%, 3%.
2. $3\frac{1}{3}\%$, 7%, $3\frac{1}{2}\%$, $2\frac{1}{2}\%$, 4%.
3. 5 years, $2\frac{1}{2}$ months, $3\frac{1}{2}$ years, 24 days, 8 months.
4. £1,022.
5. £100.
6. £198 6s. 8d.
7. £97 1s 9d.
8. £750 0s. 5d.
9. £221 15s. 10d.
10 £225

11. £4,526 13s. 4d.

12. 3·2%.

13. 3·39% (taking it to 11th March of following year).

14. $4\frac{1}{2}\%$.

15. 6·76 . . .%.

16. £75.

17. $12\frac{1}{2}\%$.

18. £671 5s.

19. £450.

20. £505 19s. 1d.

21. £600 13s. 3d., £753 9s. 9d., £1,006 6s. 5d., £964 19s., £328 16s. 11d.

22. 3·015 . . %.

23. £8 14s. 7d.

24. £453 19s. 6d.

25. £12, £758 8s. 9d.

26. £845 10s. 3d.

27. £532 10s., $6\frac{1}{4}\%$ (assuming that £500 is borrowed each time), or £533 5s. 10d., 6·658 . . . % (if interest is borrowed as well).

28. 2·88 . . .%.

CHAPTER XIII

1. 78, 156, 234, 312.
2. 2,197, 2,197, 8,788, 15,379.
3. £426 13s. 4d., £293 6s. 8d.
4. 25%, $33\frac{1}{3}\%$, $41\frac{2}{3}\%$.
5. £175, £250, £325.
6. £120, [illegible]0, £60.
7. 30, [illegible], 72.
8. £1 4s. 9d., £2 17s. 9d.
9. £400, £150, £480.
10. 152, 171, 361.
11. £70 6s. 3d., £134 6s. 6d., £55 4s. 9d., £40 2s. 6d., $4\frac{1}{2}$d.
12. £11 11s., £13 4s., £21 9s.
13. £51 8s. 9d., £34 5s. 10d., £25 14s. $4\frac{1}{2}$d.
14. B receives £1 6s. 1d.
15. 5 . 2
16. 4 lbs.

17. 18·94 . . .%.

18. 5 : 8.

19. $\frac{5}{18}$ of a pint extra water (at first acid = $\frac{1}{6}$ pint: after addition it is to be $\frac{3}{20}$ of water ∴ total of water = $\frac{1}{6} \times \frac{20}{3}$ pts.).

20. 18s.

21. £36 10s. 4d., £2,191

22. $3\frac{13}{20}$%, 4%.

23. £300 2s. 6d., £274 8s., £308 14s.

24. £600, £600, £540, £360.

25. £260 17s. 4d., £339 2s. 8d.

26. £350, £450

27. 40s., 11s. 4d.

28. 88 grs. and 2 grs.

29. 467, 467, 836.

30. £3,800, £1,200.

31. 2 : 9.

32. i. One solution is 1 : 2 : 6 : 1 : 1. ii. One solution is 1 : 3 : 5 : 1 : 1.

33. £1,054, £2,076.

34. £363, £272 5s., £190 11s. 6d.

35. $\frac{15}{38}$ o. p.

36. £2,600 0s. 1d.

37. 51d.

38. $1\frac{7}{8}$ oz., £7 19s.

CHAPTER XIV

1. 4,900, 604, 1,755, 1,568, 2,240, 348, 98, 1,528

2. $43\frac{3}{10}$, $5\frac{11}{28}$, $15\frac{75}{112}$, 14, 20, $3\frac{3}{28}$, $\frac{7}{8}$, $13\frac{9}{14}$.

3. 1,605, $3,754\frac{1}{2}$, 3,202, 126,720, 304, 487, 4,680, 1,345,949.

4. $44\frac{7}{12}$, $104\frac{7}{24}$, $88\frac{17}{18}$, 3,520, $8\frac{4}{5}$, $13\frac{13}{36}$, 130, $37,387\frac{1}{6}$

5. 10,980, 4,544, 750, $361,102\frac{1}{2}$, 21,376, $4,143\frac{1}{4}$, 118, $25,320\frac{1}{2}$, 77,166.

6. £16 15s. 11d., £7 11s. 3d., £1 10s. 5d., £841 18s. $5\frac{1}{2}$d., £4 19s., £15 14s. $10\frac{1}{2}$d., £2 1s. 8d., £4 3s. $2\frac{1}{2}$d., £26 16s. 10d., £31 14s. 6d.

7. £$3\frac{9}{16}$, £$11\frac{13}{16}$, £$\frac{11}{40}$, £$6\frac{7}{15}$, £$41\frac{3}{5}$, £$1,620\frac{59}{160}$, £$116\frac{7}{8}$, £$11\frac{11}{120}$, £$846\frac{27}{160}$, $71\frac{1}{7}$s., $236\frac{1}{4}$s., $5\frac{1}{2}$s., $129\frac{1}{3}$s., 832s., $32,407\frac{3}{8}$s., $2,337\frac{1}{2}$s., $221\frac{5}{6}$s., $16,923\frac{3}{8}$s., 855d., 2,835d., 66d., 1,552d., 9,984d., $388,888\frac{1}{2}$d., 28,050d., 2,662d., $203,080\frac{1}{2}$d.

8. 5 t. 9 c. 1 qr. 26 lbs.

9. £1 9s. $9\frac{1}{2}$d.
10. 945 sq. ft., 15 sq. in.
11. 133, 1,995, 7,600, 10·2625, $12\frac{3}{4}$, $\frac{1}{24}$, 585, $13\frac{1}{8}$
12. $\frac{71}{750}$
13. 15 yds., 45 yds.
14. 25 b. 1 p. 1 g.
15. 357,120
16. $42\frac{1}{2}$ sq. yds., $56\frac{2}{3}$ yds.
17. $62\frac{1}{8}$
18. 2,220.
19. 11.
20. $1\frac{1}{4}$ ft.
21. 576, £2 14s.
22. £11 11s. 9d.
23. 121 lbs.
24. 4·5 in.
25. 132,108 in.
26. 19 ft. 6 in.
27. ·3042 cu. ft.
28. $17\frac{1}{2}$ cwt.
29. 1,008 sq. ft., £1 16s.
30. 338 ft. 4 in.
31. £77 3s. 6d.
32. 46.
33. 279,092
34. £9 19s. 9d.
35. 11 pieces, 16s. $0\frac{1}{2}$d.
36. $13\frac{19}{128}$.
37. 420,000

CHAPTER XV

1. 13·08 kg.
2. 6·16.
3. 208.
4. £135 11s. 6d.
5. 360 grains.
6. 38.

7. 70 cents.

8. 25 min.

9. 2,915 metres.

10. 12s.

11. 10·7584.

12. 53 cm.

13. 5s. $5\frac{1}{2}$d.

14. 25%.

15. 370.

16. 50 cm.

17. 5,346.

18. 1,202.

19. $416\frac{2}{3}$ days.

20. 612.

21. 5.

22. 9.

23. 1·02.

24. 1,033·466, 438·539, 109·088, 13,150·714, 7·219, 3,280·843

25 640·08, 80·47, 114·30, 24·69, 61·72, 73·84, 92·51, 22·40, 40·46, 34·52, 7·43.

26 36, 1,775, 207, 30, 1, 195, 1,034.

27. $156\frac{1}{4}$.

28. 420,000.

29. 114 cm.

30. 1 fr. 80.

31. 325.

32. 145.

33. 1,200 c.c.

34. 191·52.

35. 157 frs. 24.

36. 44 frs. 32

37. 1,841 millions.

38. 756·60, 22,992·32, 237·84, 22·54, 15·76, 594·46, 25,220, £793 0s. $5\frac{1}{4}$d., £7 5s. $6\frac{1}{4}$d., £159 9s. $4\frac{1}{4}$d., £25 2s. $3\frac{1}{4}$d, £1,795 5s. $0\frac{1}{2}$d.

39. 3 crores 23 lacs 10,000 rupees, 65 lacs.

40. £5 2s. $2\frac{1}{2}$d

41. £6,666 13s. 4d.

THE END